

**SAMPLE TEST # 3**

Solve the following exercises. **Show your work.**

**Ex. 1.** Evaluate the integrals:

(a)  $\int_0^1 \int_0^\pi \frac{1}{x+1} + \sin y \, dy \, dx =$

(b)  $\int_{-1}^2 \int_{-y}^{y+2} (x + 2y^2) \, dx \, dy =$

(c)  $\int_{-2}^0 \int_0^{\sqrt{4-y^2}} \frac{dy \, dx}{\sqrt{9-x^2-y^2}} =$

(d)  $\int_0^1 \int_y^1 e^{-x^2} \, dx \, dy =$

**Ex. 2.** Find the mass of the solid bounded by the hemisphere  $x^2 + y^2 + z^2 \leq R^2$ ,  $z \geq 0$ , with the density  $\delta(x, y, z) = x^2 + y^2 + z^2$ .

**Ex. 3.** Find the mass of the plane lamina bounded by  $x = 0$  and  $x = 9 - y^2$  with density  $\delta(x, y) = x^2$ .

**Ex. 4.** Find the area of the surface of the graph of  $z = 1 + 2x + 2y$  that lies above the region bounded by parabolas  $y = x^2$  and  $x = y^2$ .

**Ex. 5.** Set up the integral formulas, **including the limits of the integrations**, for the following problems. *Do not evaluate the integrals!*

(a) The moment of inertia around the  $z$ -axis of the solid  $T$  with the density  $\delta(x, y, z) = x^2 + e^z$  bounded by the surfaces:  $6x + 2y + z = 12$ ,  $x = 0$ ,  $y = 0$ , and  $z = 0$ .

(b) The volume of the solid bounded by  $z = x^2 + y^2$ ,  $z = 0$ ,  $x = 0$ ,  $y = 0$ , and  $x + y = 1$ .

(c) The center of the mass of the plane lamina bounded by  $y = x^2$  and  $y = 2x + 3$ , with the density  $\delta(x, y) = x^2$ .

(d) The area of the region inside  $r = \sin \theta + 2.5$  and outside  $r = 3$ .