

MATH 251
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Spring 2003

SAMPLE TEST # 1

Solve the following exercises. **Show your work.**

Ex. 1. Write the parametric equations of the line that passes through the point $P(11, 13, -7)$ and is perpendicular to the plane with the equation: $x - 2z = 17$.

Ex. 2. Find the volume of the pyramid with the vertices: $P(3, 2, -1)$, $Q(-2, 5, 1)$, $R(2, 1, 5)$, and the origin $O(0, 0, 0)$.

Ex. 3. Describe in geometric terms the graph of the following equation. Be specific.

$$2x^2 + 2y^2 + 2z^2 = 7x + 9y + 11z.$$

Ex. 4. Find the curvature κ of the curve with position vector $\mathbf{r}(t) = \mathbf{i} \cos t + \mathbf{j} \sin t + 2t \mathbf{k}$.

Ex. 5. Let $\mathbf{a} = \langle 0, 1, 2 \rangle$, $\mathbf{b} = \langle -1, 0, 7 \rangle$, and $\mathbf{c} = \langle 2, 3, -1 \rangle$. Evaluate: $2\mathbf{a} - \mathbf{b} + \mathbf{c}$, $|\mathbf{c}|$, and $(\mathbf{a} \cdot \mathbf{b}) \cdot (\mathbf{b} \times \mathbf{c})$.

Ex. 6. Let $\mathbf{v}(t) = \mathbf{i}(t + 1)^{-1} + \mathbf{k}t^3$ be a velocity of a particle. Find the acceleration vector $\mathbf{a}(t)$ of the particle and its position vector $\mathbf{r}(t)$, where its initial position was $\mathbf{r}_0 = 3\mathbf{i}$.

Ex. 7. Find the the arc length, s , of the curve with position vector $\mathbf{r}(t) = 2e^t \mathbf{i} + 2t \mathbf{j} + e^{-t} \mathbf{k}$ from $t = 0$ to $t = 1$.

Ex. 8. State and graph the largest possible domain of the function $h(x, y) = \sqrt{y - x^2}$.

Ex. 9. Describe and sketch the graph of the equation: $4z^2 = x^2 + y^2$.

Ex. 10. Convert the equation: $x^2 + y^2 + 2z^2 = 1$ to the cylindrical and spherical coordinates.