SAMPLE TEST # 2

1. Find the intervals on which function \( f(x) \) is increasing, and those on which it is decreasing.
\[
  f(x) = x^4 + 8x^3 + 7
\]

2. Apply the first derivative test to find and classify (as local minimum, local maximum, or not an extremum) each of the critical points of
\[
  h(x) = x^4 - 2x^2 + 100
\]

3. Find the third derivative \( g'''(x) \) of \( g(x) = x^4 + x^{-1} + \cos x \).

4. Find \( y' \) and \( y'' \) assuming that \( y \) is defined implicitly as function of \( x \) by the equation
\[
  \sin^2 x + \cos^2 y = 1.
\]

5. Sketch the graph of the function \( f(x) = 3x^5 - 5x^3 \) indicating critical points and inflection points. Apply the second derivative test at each critical point.

6. Find the particular solution of the differential equation \( \frac{dy}{dx} = 4x^3 + x \) that satisfies the initial condition \( y(-1) = 2 \).

7. Find the most general antiderivatives of
   (a) \( g(z) = z^{1/2} + 5 \)
   (b) \( f(x) = \cos(2x + 1) \)

8. Evaluate the integrals:
   (a) \( \int \left( \sqrt{x} - \frac{2}{\sqrt{x}} \right) \, dx = \)
   (b) \( \int \left( \frac{1}{2x^2} + x^3 \right) \, dx = \)

9. Evaluate the limit
\[
  \lim_{x \to \infty} \frac{\sqrt[3]{x^6} + x - 1}{x^2 + 5} = \]