SAMPLE TEST # 3

Show your work.

1. Let \( f(x) = x^4 - 6x^2 \) be on the interval \([-2, 3]\).
   
   (a) Find all points at which \( f \) has local maximum or a local minimum. Show your work.
   
   (b) Find all points at which \( f \) has an inflection point. Show your work.
   
   (c) Graph \( f \) using (a) and (b). Mark on the graph: all local extrema, inflection points, concavity, and \( x, y \)-intercepts.

2. Assume that \( f(3) = 7 \) and that \( f'(x) \geq 5 \) for all values of \( x \). Use Mean Value Theorem to determine what is the smallest possible value of \( f(6) \).

3. Find the limits.
   
   (a) \( \lim_{x \to -\infty} \left( x + \sqrt{x^2 + 2x} \right) \)
   
   (b) \( \lim_{x \to \infty} \frac{\sqrt{x^2 - 9}}{2x - 6} \)

4. Find the horizontal and vertical asymptotes of the function \( f(x) = \frac{x^3 + x^2 - 1}{2x^3 + x} \).

5. Find the point on the line \( 2x + 3y + 5 = 0 \) that is closest to the point \((-1, -2)\).

6. The sum of two nonnegative numbers is 10. Find the minimum possible value of the sum of their cubes.

7. Sketch the graph of the function that satisfies all of the given conditions.
   
   \( f'(2) = 0, \ f'(0) = 1, \)
   
   \( f'(x) > 0 \) if \( 0 < x < 2, \ f'(x) < 0 \) if \( x > 2, \)
   
   \( f''(x) < 0 \) if \( 0 < x < 4, \ f''(x) > 0 \) if \( x > 4, \)
   
   \( \lim_{x \to -\infty} f(x) = 0, \)
   
   \( f(-x) = -f(x) \) for all \( x. \)

8. Use the second derivative test to find local maxima and local minima of the function \( f(x) = x^3 - 3x + 1 \).

9. Find absolute maximum and absolute minimum of \( f(x) = 2x^3 + x^2 - x + 1 \) on \([0, 2]\).