

SAMPLE FINAL TEST

1. Use the Limit Law(s) to find the following limits. Show all steps.

• $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - 3x + 2} =$

• $\lim_{x \rightarrow 1} \frac{2 - \sqrt{5 - x}}{\sqrt{x} - 1} =$

• $\lim_{x \rightarrow 4^+} \frac{x^2 - 16}{|4 - x|} =$

2. Let $f(x) = x^4 - 2x^2 + 17$. Find $f'(x)$ using the limit definition for $f'(x)$. (Do not use differentiation formulas.) Show your work.

3. Find $f'(x)$ using the differentiation formulas (not the limit definition for $f'(f)$). Do not simplify your answer.

• $f(x) = 2x^{99} + \frac{1}{\sqrt[3]{x}} - \sin 1$

• $f(x) = (x^{20} + 17x^3 - x^{-3} + 17)(x^5 - x^4 + x + 1)$

• $f(x) = \frac{\sqrt{x} - 1}{\sqrt{x} + 1}$

4. Find the equation of the tangent line to the curve $y = \sqrt{x}$ at the point $(4, 2)$. Use differentiation formulas to find the derivative.

5. Find the derivatives of the following functions. Do not simplify your answer.

(a) $f(x) = (x^3 - 12x^2 + 5)^{33}$

(b) $g(x) = \tan(x^3 - x + 17)$

(c) $f(y) = y^2 \cot(y^5)$

6. Find y' by implicit differentiation when $\frac{y}{x - y} = x^2 + 1$.

7. Use linear approximation to approximate the value of $\sqrt[3]{28}$.

8. Find the limit
 $\lim_{x \rightarrow 0} \tan 3x \cot 5x$.

9. Let $f(x) = x^4 - 6x^2$.

(a) Find all points at which f has local maximum or a local minimum. Show your work.

(b) Find all points at which f has an inflection point. Show your work.

(c) Graph f using (a) and (b). Mark on the graph: all local extrema, inflection points, concavity, and x, y -intercepts.

10. Find the limit.

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x^2 - 9}}{2x - 6}$$

11. Find the horizontal and vertical asymptotes of the function $f(x) = \frac{x^3 + x^2 - 1}{2x^3 + x}$.

12. Use the second derivative test to find local maxima and local minima of the function $f(x) = x^3 - 3x + 1$.

13. Find absolute maximum and absolute minimum of $f(x) = 2x^3 + x^2 - x + 1$ on $[0, 2]$.

14. Evaluate the following integrals.

(a) $\int (3x^4 - 5x + 3) dx$

(b) $\int \frac{2x - 3x\sqrt[3]{x} - 1}{x^4} dx$

(c) $\int \cos 5x dx$

(d) $\int_1^2 5x(x + 3x^2) dx$

(e) $\int_{-1}^2 (x + |x|) dx$

(f) $\int 4x^7 \sqrt{2 + x^4} dx$

15. Sketch the area bounded by the following curves.

$$x = 0, \quad y = 0, \quad y = x^3 - 1, \quad x = 3$$

Evaluate the area.

16. Find the volume of the solid obtained by rotating the region bounded by the curves $y = x^2$ and $y = x^5$ about the x -axis.

17. When a particle is at the distance x meters from the origin, a force of $\cos(\pi x/3)$ newtons acts on it. How much work is done in moving the particle from $x = 1$ to $x = 2$?

18. Find the average value of the function $f(x) = \cos x$ over the interval $[\pi/2, \pi, 4]$.