SAMPLE TEST # 4

Solve the following exercises. **Show your work.** (No credit will be given for an answer with no supporting work shown.)

**Ex. 1.** Transform the following system of equations into a single second order equation in terms of $x_1$. Then give the initial condition for the resulted equation that corresponds to the given initial conditions. Do not solve.

$$x_1' = -0.5x_1 + 2x_2; \quad x_2' = -2x_1 - 0.5x_2; \quad x_1(0) = -2, \quad x_2(0) = 2.$$ 

**Ex. 2.** Use eigenvalues and eigenvectors to find the general solution of the given systems of differential equations. The solution must be expressed in terms of real-valued functions.

(a) $x' = \begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix} x$

(b) $x' = \begin{pmatrix} 1 & 2 \\ -5 & -1 \end{pmatrix} x$

(c) $x' = \begin{pmatrix} 6 & -3 \\ 3 & 0 \end{pmatrix} x$

**Ex. 3.** Solve the following boundary value problem or show that it does not have a solution.

$$y'' + 4y = 0, \quad y(0) = 0, \quad y(\pi) = 0.$$ 

**Ex. 4.** Determine whether the method of separation of variables can be used to replace the partial differential equation $u_{xx} + u_{xt} + u_t = 0$ by a pair of ordinary differential equations. If so, find the ordinary differential equations. Do not solve them.

**Ex. 5.** Solve the heat equation: $u_t = 9u_{xx}, \quad u(0,t) = u(2,t) = 0, \quad u(x,0) = 13$ for $0 < x < 2$. 