Math 641 **Homework 1** **Spring 2007**

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1. If A is an abelian group and $n > 0$ is an integer such that $na = 0$ for all $a \in A$, then A is a $\mathbb{Z}_n$-module, with action of $\mathbb{Z}_n$ on A given by $\bar{k}a = ka$, where $k \in \mathbb{Z}$ and $k \mapsto \bar{k} \in \mathbb{Z}_n$ under the canonical projection $\mathbb{Z} \to \mathbb{Z}_n$.

2. Let $f : A \to B$ be an R-module homomorphism.

(a) f is a monomorphism if and only if for every pair of R-module homomorphisms $g, h : D \to A$ such that $fg = fh$, we have $g = h$.

(b) f is an epimorphism if and only if for every pair of R-module homomorphisms $k, t : B \to C$ such that $kf = tf$, we have $k = t$.

3. Let I be a left ideal of a ring R and A be an R-module.

(a) If S is a nonempty subset of A, then

$$IS = \left\{ \sum_{i=1}^{n} r_i a_i \mid n \in \mathbb{N}^*,\ r_i \in I,\ a_i \in S \right\}$$

is a submodule of A.

(b) If I is a two-sided ideal, then A/IA is an (R/I)-module with scalar multiplication given by $(r + I)(a + IA) = ra + IA$.