Math 283 **Homework** **Spring 2007**

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1. Consider the following predicates with one variable. Find the truth set for each of them assuming the the universe of discourse is the set $\mathbb{R}$ of real numbers.

(a) $(x < 2) \wedge (x > 1)$

(b) $(x > 3) \vee (x < 0)$

(c) $(x < 5) \Rightarrow (x < 2)$

(d) $(x < 9) \Leftrightarrow (x > 12)$

(e) $\sim (x < 10) \wedge (x < 20) \Leftrightarrow (x > 17)$

2. Consider the following predicates with two variables. Assume that for both x and y the universe of discourse is the set $\mathbb{R}$ of real numbers. Draw the truth set on the xy-plane.

(a) $(x \leq y) \wedge (x \geq y - 2)$

(b) $(y \geq x^2 - 1) \wedge (y \leq x^2 + 1)$

(c) $(x^2 + y^2 \leq 4) \Rightarrow (y \leq 0)$

(d) $(y \geq x^2) \Leftrightarrow (x^2 + y^2 \leq 1) \vee (x^2 + y^2 \geq 9)$

3. Identify correct and incorect theorems. For incorrect theorems give valuation of the variables that shows that they are incorrect.

(a) **Theorem?** Let x and y be real numbers. Suppose $x < 6$ and $y < 10$. Then $x + y < 16$.

(b) **Theorem?** Let x and y be real numbers. Suppose $xy > 6$. Then $x < 3$ implies that $y > 2$.

(c) **Theorem?** Let x and y be real numbers. Suppose that $xy > 6$ and $x > 0$. Then $x < 3$ implies that $y > 2$.

(d) **Theorem?** Let x and y be real numbers. Suppose that $x + y < 10$ or $x < 5$. Then $x > 3$ implies that $y < 7$.

(e) **Theorem?** Let x and y be real numbers. Suppose that $x < 0$ implies that $y < 0$. Then $xy \geq 0$.

(f) **Theorem?** Let x and y be real numbers. Suppose that $x < 0$ if and only if $y < 0$. Then $xy \geq 0$.