TO MY COLLEAGUES IN MATHEMATICS

Have you ever seen any expansion resembling the following?

Special expansion of a function \( F \) by means of a function \( f \):

\[
F(x) = \sum_{n=0}^{\infty} (-1)^n \frac{A_n}{n!} D_x^n f(x)
\]

where

\[
A_n = \sum_{k=0}^{n} \left( \begin{array}{c} n \\ k \end{array} \right) D_t^k \left( \int_{-\infty}^{+\infty} \frac{1}{f(x) e^{tx}} dx \right) \bigg|_{t=0}^{+\infty} \int_{-\infty}^{+\infty} x^{n-k} F(x) \, dx
\]

Can you determine any conditions sufficient to allow such an expansion?

I ran into this expansion many years ago in a statistics journal, I see from my old notes, where I had written that the author attributed it to A. C. Aitken (Univ. of Edinburgh). However a few years before his death I asked Aitken about it in a letter and he denied any association with it and did not volunteer any remarks to shed light on it. I have, unfortunately, been unable to determine where I ran into the expansion. It seems to be unusual and I offer it to you for study. I have mentioned it to several other colleagues in letters recently, but no one has added anything. I would like to determine what functions, \( F \), allow such an expansion using another function, \( f \).

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