Bell and Catalan Numbers: A Research Bibliography of Two Special Number Sequences

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Chapter 1

Introduction and Discussion

Below we present what we believe to be fairly complete bibliographies up the year 1979 for two interesting combinatorial number sequences:

The Catalan sequence: $1, 1, 2, 5, 14, 42, 132, 429, 1430, \ldots$
The Bell sequence: $1, 1, 2, 5, 15, 52, 203, 877, 4140, \ldots$

We shall designate terms in the two sequences by $C(n)$ and $B(n)$ respectively, with $n = 0, 1, 2, 3, \ldots$ These sequences arise in a variety of novel combinatorial situations, as will be seen from a perusal of the papers and books in our bibliographies.

Explicit formulas are:

$$C(n) = \frac{1}{n+1}\binom{2n}{n} = \frac{2n!}{n!(n+1)!}, \tag{1.1}$$

$$B(n) = \frac{1}{e}\sum_{k=0}^{\infty}\frac{k^n}{k!} \quad \text{Dobiński's formula.} \tag{1.2}$$

$$B(n) = \sum_{k=0}^{n} S(n,k), \tag{1.3}$$

Here, S(n,k) denotes a Stirling number of the second kind, where

$$S(n,k) = \tfrac{1}{k!}\Delta^k 0^n = \tfrac{1}{k!}\textstyle\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}j^n$$

$$B(n+1) = \sum_{k=0}^{n}\binom{n}{k}B(k) \text{ , with } B(0) = 1. \quad \text{Recurrence.} \tag{1.4}$$

$$C(n) = \sum_{k=1}^{n} C(k-1)C(n-k), \text{ with } C(0) = 1. \quad \text{Recurrence.} \tag{1.5}$$

$$C(n+1) = \sum_{k=1}^{n} C(k)C(n-k) \text{ , with } C(0) = 1. \quad \text{Alternative recurrence} \tag{1.6}$$

$$\mathbf{B}(x) = e^{e^x - 1} = \sum_{n=0}^{\infty} B(n) \frac{x^n}{n!} \quad \text{Generating function.} \tag{1.7}$$

$$\mathbf{C}(x) = \frac{1 - \sqrt{1-4x}}{2x} = \sum_{n=0}^{\infty} C(n)x^n \quad \text{Generating function.} \tag{1.8}$$

$$x\mathbf{C}^2(x) = \mathbf{C}(x) - 1 \quad \text{Functional relation} \tag{1.9}$$

$$\sum_{\substack{1 \le a_1 \le a_2 \le \ldots \le a_n \\ a_i \le i}} 1 = C(n) \quad \text{Enumeration formula.} \tag{1.10}$$

Combinatorial interpretations of the numbers are plentiful. For example, $C(n-1)$ = the number of ways of associating n elements in forming partial products. Example: a(b(cd)), a((bc)d), (ab)(cd), ((ab)c)d, and (a(bc))d are the only 5 = C(3) ways of associating 4 elements in multiplying out by partial products. It is easy to confound the two sequences because of the similar values 1, 1, 2, 5, and 14 versus 1, 1, 2, 5, and 15. Thus, we have that B(n) = the number of ways of factoring a product of n distinct primes. Example: (abc), (a)(bc), (b)(ac), (c)(ab), (a)(b)(c) are the only 5 = B(3) ways, in the case of three primes. In some cases of complexity, one can easily mistake 14 for 15 at the next step! Thus, as far as the writer can determine, Rota, in his bibliography on the exponential (= Bell) numbers, gave a reference to a paper of Catalan which seems to have to do with the Catalan numbers and not the Bell numbers. Another way to define the Bell numbers combinatorially is this: B(n) = the number of distinct equivalence relations connecting n elements.

Two recent historical papers are given in the bibliographies which follow: a paper of Rota on Bell numbers, and a paper of W. G. Brown on the Catalan numbers. The Catalan numbers seem to trace back to Euler, and were also studied by Fuss, Segner, and many others. The Bell numbers are named for Eric Temple Bell, who did much to popularize them and generalize them. He says that Euler is often credited with their discovery, but no specific reference in any of Euler's work has been shown. The writer has found that the Bell numbers were known to Christian Kramp in 1796. Also, Dobiński's formula (1877) was not only possibly known to Kramp, but also appears in the Matematicheskii Sbornik as a problem in Vol. 3(1868), p.62, very explicitly. It is hoped that this present bibliography will shed some light on the subject, containing, as it does, 470 items about the Catalan numbers and 202 about the Bell numbers. More information exists in the author's card file, but it is entirely out of the question to present this without doing a major book a là Leonard Eugene Dickson (c.e. his monumental History of the Theory of Numbers. Let us indicate an item in the Catalan bibliography by prefixing the letter C before a number and for the Bell number bibliography by prefixing the letter B.

In items C141 through C153 will be found a very brief indication of the vast literature concerning the coefficients defined by $A_k(a,b) = \frac{a}{a+bk}\binom{a+bk}{k}$, which include the Catalan numbers for a =1 and b = 2. The reviews noted in C147 will give considerable information about formulas such as

$$\sum_{k=0}^{n} A_k(a,b)A_{n-k}(c,b) = A_n(a+c,b) \tag{1.11}$$

which traces back to H.A. Rothe in 1793 and which reduces to (1.5) above for $a = c = 1$, $b = 2$. Over 500 items bearing on this and the related formula of Abel

$$\sum_{k=0}^{n} B_k(a,b)B_{n-k}(c,d) = B_n(a+c,b) \tag{1.12}$$

$$(B_n(a,b) = \frac{a}{a+bk}\frac{(a+bk)^k}{k!})$$

could be listed! Formulas like these turn up frequently in graph theory.

Likewise, information about papers treating functional expansions and special functions and polynomial systems has been omitted here. Exceptions are Bell's basic paper B16 and the author's work B72. Over 150 papers in this direction could be mentioned.

The present bibliographies were announced in B73 (=C150), before it was decided just how much detail to give, or how to coordinate the work.

The author would appreciate any further references anyone thinks ought to be cited, as he does not guarantee he has included every reference.

The number $B(n)$ enumerates the partitions of a set of n elements, e.g. the number of different rhyme schemes for a verse of n lines.

Becker in C21 remarked that there are more than 40 different combinatorial interpretations of the Catalan sequence. One of the challenging problems is to show how to construct isomorphisms between these seemingly different structures. Here is an illustration of the five Catalan parenthesizations for a product of four elements ABCD, where the order of the elements is unchanged:

(AB)(CD), ((AB)C)D, A(B(CD), (A(BC))D, A((BC)D),

For five elements there are 14 parenthesizations. By contrast, the Bell numbers enumerate the number of ways a set of n elements may be partitioned into disjoint sets. For example, a set of three elements has five possible partitionings:

(ABC), (A, BC), (B, AC), (C, AB), (A, B, C, D)

For a set of four elements there are 15 partitionings.

Another aspect of the Catalan sequence is the question of arithmetic properties. Here we have included quite a number of important items. Combinatorially, it is clear that $n+1$ divides $\binom{2n}{n}$ since this ratio turns out to count the number of ways of doing various things. But it is also trivial to see that $\frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n-1}$ which proves that the ratio is indeed an integer. Let us define a generalized binomial coefficient in terms of an arbitrary sequence A_n with $A_0 = 0$ and $A_n \neq 0$ for $n \geq 1$, by writing

$$\binom{n}{k}_{A_n} = \frac{A_n A_{n-1} \dots A_{n-k+1}}{A_k A_{k-1} \dots A_1} \tag{1.13}$$

with $\binom{n}{0}_{A_n} = 1$.

In C151 and C154, the author has shown how we may obtain analogs of the Catalan numbers and it is proved there that in the case when $A_n = F_n =$ a Fibonacci number defined by $F_{n+1} = F_n + F_{n-1}$, $F_0 = 0$, $F_1 = 1$, it is true that

$$F_{n+1} \mid \binom{2n}{n}_{A_n}. \tag{1.14}$$

Suitable combinatorial interpretations are now being investigated. "Shaking hands across the table" might be an amusing way to describe the problem of connecting $2n$ points on a circle in pairs. If arms do <u>not</u> cross (that is to say, chords do not intersect), then the Catalan numbers enumerate the number of ways.

Let $P_k(n)$ denote the number of ways in which $2n$ points around a circle may be joined in pairs with k intersections of chords. Then $P_0(n) = C(n) = \frac{\binom{2n}{n}}{n+1}$. In our example, $n = 3$, so $P_0(3) = 5$. John Riordan, item C310, has shown how to determine $P_k(n)$ in general, using ideas from the work of Touchard, who did not explain his results in this way, but who, nevertheless, had the solution implicitly. The total number of ways of joining the pairs of points at all is $\sum_{k=0}^{\frac{n(n-1)}{2}} P_k(n) = 1 \cdot 3 \cdot 5 \cdots (2n-1)$. For $n = 3$ the total is $1 \cdot 3 \cdot 5 = 15$. There are then 6 cases with just one crossing, 3 cases with two intersections, and 1 case with three intersections. The numbers $P_k(n)$ have a number of interestin properties.

The oldest reference I have found to a case of a combinatorial study involving Bell numbers (and by implication, as a subset, the Catalan numbers) is in some diagrams introduced by the old Japanese mathematicians (wasan) to editions of the famous <u>Tale of Genji</u> by Lady Shikibu Murasaki (fl. 978 - 1031?) in the seventeenth and eighteenth centuries.

Lady Murasaki was the Japanese novelist who may well be called the Shakespeare of Japan. <u>Tale of Genji</u> dates to about 1000 A.D. In the original, there are 54 chapters. The middle 52 chapters have designs at the top of each chapter showing the possible arrangements of 5 different incense sticks which may be all the same color or any combination of 5 different colors allowing repetition. It is not difficult to see that for n sticks the number of possible diagrams is $B(n)$, the number of partitions of a set of n things.

The horizontal bar in the diagrams connects those sticks of the same color. A discussion of this combinatorial stucture is given by Tsuruichi Hayashi in

Japanese in item B82 (=C183) Unfortunately, the English editions [1] in our libraries fail to carry these interesting diagrams so that the mathematical significance of the 52 chapters is lost to Western readers. It is also unfortunate that Hayashi's paper seems to be the only reference discussing this, and reading the old Japanese is not at all easy. If we omit all the diagrams where lines cross, i.e. diagrams such numbers 4, 5, and 6 in our sample above, then it appears that the number of diagrams we have left is a Catalan number. Of the 52 original Murasaki diagrams, 10 have crossed lines and so the subset has 42. In the case $n = 4$, there are, of course, 15 Murasaki diagrams. One of these, and only one, has crossed lines.

Martin Gardner, C133(=B167), notes that Joanne Growney, C162 (=B169), has proved that this remarkable property is indeed true in general.

It is certainly remarkable that a combinatorial structure had something to do with a literary work as early as the 1600's. One is tempted to refer to Murasaki structures in honor of Lady Murasaki. As with the ordering of the 64 hexagrams in the I Ching, it is interesting to speculate on why Wasanists arranged the 52 diagrams in the particular order chosen. There is a very brief review in German of Hayashi's 1931 article, in JFdM, 57(1931), 20. I have found no other discussion of the subject. A translation of Hayashi's discussion is being made by Sin Hitotumatu in Japan (1978). Harry Vandiver (On the desirability of publishing classified bibliographies of the mathematics literature, American Mathematical Monthly, 67(1960), January, 47-50) made an appeal to mathematicians to do more in the way of information retrieval, and it is in this spirit that the present work is offered.

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The biographical reference numbers in Kuchinski's thesis accord with the 1976 printing of the bibliographies and thereby differ from the bibliographies below which have been re-alphabetized.

Remark: In preparing this new version of the bibliographies, Mr. Glatzer was supported by a stipend from the Department of Mathematics, West Virginia University.

The totals in the present listing are approximately 200 Bell references and 470 Catalan references.

[1]E.g. Arthur Waley's translation published in the Modern Library series

A FINAL NOTE

A valuable reference on Catalan numbers is a splendid article by Martin Gardner (decd.), C133 (= B167), in the June 1976 Scientific American magazine. Gardner has followed this with an equally interesting one on Bell numbers, B186, in the May 1978 issue.

We end by dedicating our Bell bibliography to the memory of Lady Murasaki.

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Chapter 2

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Chapter 3

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