

## 3.1 The Product and Quotient Rules

- Question: How do you compute the derivative of the product of two functions?
  - Sometimes, just multiply them out

$$\text{Ex: } h(x) = (x + 1)(x - 1) = x^2 - 1$$

$$h'(x) = \frac{d}{dx}(x^2 - 1) = 2x$$

# Multiplying May Be Complicated

- Ex:  $h(x) = (x+1)(x-1)^{10}$
- Ex:  $h(x) = (x^2-2x-1)(x^3+10x^2+x-10)$
- In each case above, multiplying out these expressions will be very time consuming

# The Product Rule

If  $h(x)$  is the product of two functions, say  $h(x)=f(x)g(x)$ , then

$$\frac{d}{dx} f(x)g(x) = f'(x)g(x) + f(x)g'(x)$$

# An Example

- Ex:  $h(x) = (x+1)(x-1)^{10}$

$$\begin{aligned}h'(x) &= \left[\frac{d}{dx}(x+1)\right](x-1)^{10} + (x+1)\left[\frac{d}{dx}(x-1)^{10}\right] \\ &= (1)(x-1)^{10} + (x+1)[10(x-1)^9(1)] \\ &= (x-1)^{10} + 10(x+1)(x-1)^9\end{aligned}$$

# Another Example

- Ex:  $h(x) = (x^2 - 2x - 1)(x^3 + 10x^2 + x - 10)$

$$\begin{aligned}h'(x) &= (x^2 - 2x - 1)'(x^3 + 10x^2 + x - 10) \\ &\quad + (x^2 - 2x - 1)(x^3 + 10x^2 + x - 10)' \\ &= (2x - 2)(x^3 + 10x^2 + x - 10) \\ &\quad + (x^2 - 2x - 1)(3x^2 + 20x + 1)\end{aligned}$$

# Once More

- Ex:  $h(x) = (x^3 - 2x)^4(2x + 3)^5$

$$\begin{aligned}h'(x) &= \frac{d}{dx}(x^3 - 2x)^4(2x + 3)^5 \\&\quad + (x^3 - 2x)^4 \frac{d}{dx}(2x + 3)^5 \\&= [4(x^3 - 2x)^3(3x^2 - 2)](2x + 3)^5 \\&\quad + (x^3 - 2x)^4 [5(2x + 3)^4(2)] \\&= (12x^2 - 8)(x^3 - 2x)^3(2x + 3)^5 \\&\quad + 10(x^3 - 2x)^4(2x + 3)^4\end{aligned}$$

# What About Quotients?

- Quotients are a similar situation except that it's not easy to “multiply it out”, even in the simplest cases

- Ex:

$$h(x) = \frac{x + 1}{x - 1}$$

- These *can* be simplified using “partial fractions”

# The Quotient Rule

- If  $h(x) = \frac{f(x)}{g(x)}$ , then the derivative  $h'(x)$  is

$$h'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{(g(x))^2}$$

# An Example

- **Ex:** 
$$h(x) = \frac{x^2 + 3x}{x^3 - 2x^2}$$
$$h'(x) = \frac{(x^2 + 3x)'(x^3 - 2x^2) - (x^2 + 3x)(x^3 - 2x^2)'}{(x^3 - 2x^2)^2}$$
$$= \frac{(2x + 3)(x^3 - 2x^2) - (x^2 + 3x)(3x^2 - 4x)}{(x^3 - 2x^2)^2}$$

# Another Example

- Ex:

$$h(x) = \frac{(2x + 1)^5}{(1 - x)^2}$$

$$\begin{aligned} h'(x) &= \frac{\frac{d}{dx}(2x + 1)^5(1 - x)^2 - (2x + 1)^5 \frac{d}{dx}(1 - x)^2}{(1 - x)^4} \\ &= \frac{[5(2x + 1)^4](2)(1 - x)^2 - (2x + 1)^5[2(1 - x)^1](-1)}{(1 - x)^4} \\ &= \frac{10(2x + 1)^4(1 - x)^2 + 2(2x + 1)^5(1 - x)}{(1 - x)^4} \end{aligned}$$