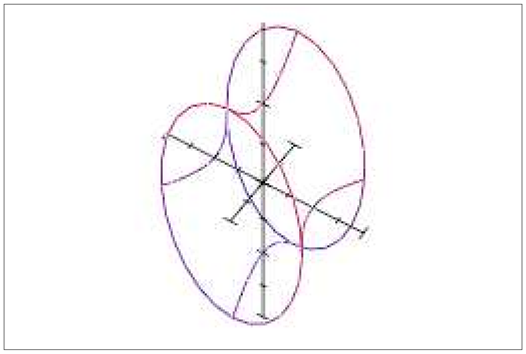
Math 251 Quiz 6

1. Sketch the graphs of the following surfaces, including any additional information as requested. You can add to the supplied axes as you need to.

a) $-x^2 + y^2 + \frac{z^2}{4} = 1$ Classify the surface, sketch representative cross sections for $x = k$ and the cross-section $z = 0$.



This is a hyperboloid of one sheet down the x axis

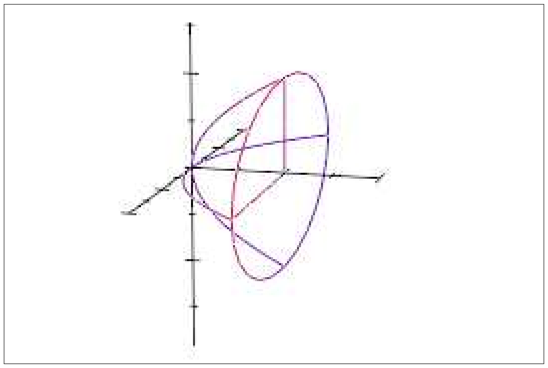
Cross-sections $x = k$ are ellipses: $y^2 + \frac{z^2}{4} = 1 + k^2$

Ellipses are twice as "tall" in z as they are wide in y

$z = 0$ is a hyperbolic cross-section: $-x^2 + y^2 = 1$

Intercepts: $y = \pm 1$, $z = \pm 4$

b) $y = \frac{x^2}{4} + z^2$ Classify the surface, sketch a representative cross section for $y = k$ and the cross section for $z = 0$.



This is an elliptic paraboloid down the positive y axis

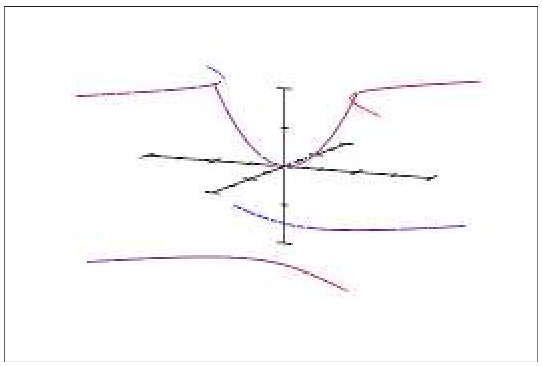
Cross-sections for $y = k$ are ellipses

Elliptical cross sections are twice as wide in x as tall in z

$z = 0$ cross section is $y = \frac{x^2}{4}$, a parabola

Intercepts: $(0,0,0)$

c) $z = -x^2 + y^2$ Classify the surface, sketch $x = 0$ cross section, $z = 1$ cross section and $z = -1$ cross section.



This is a hyperbolic paraboloid (saddle)

Sitting in the saddle, you face down the y axis

$x = 0$ cross-section: $z = y^2$

$z = 1$: $1 = -x^2 + y^2$, $z = 1 = x^2 - y^2$

2. For the curve $\vec{r}(t) = \left\langle t, t^2, \frac{1}{t} \right\rangle$, $t > 0$

a) Calculate $\vec{v} = \frac{d\vec{r}}{dt}$ and $\vec{a} = \frac{d\vec{v}}{dt}$

$\vec{v} = \left\langle 1, 2t, -\frac{1}{t^2} \right\rangle$, $\vec{a} = \left\langle 0, 2, \frac{2}{t^3} \right\rangle$

b) Find a vector tangent to the curve at $t = 1,$ the unit tangent vector $\vec{T}$ at $t = 1,$ and the equation of the line tangent to the curve at $t = 1$.

At $t = 1$ we have $\vec{v} = \langle 1, 2, -1 \rangle$ which is tangent to the curve at $t = 1$, $\vec{r} = \langle 1, 1, 1 \rangle$

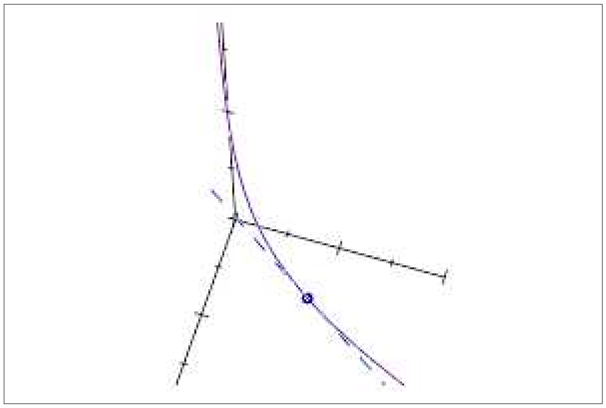
The unit tangent is $\vec{T} = \frac{\vec{v}}{\|\vec{v}\|} = \frac{1}{\sqrt{6}}\langle 1, 2, -1 \rangle$

Tangent line: Line goes through $(1, 1, 1)$ with direction $\vec{v} = \langle 1, 2, -1 \rangle$ so equation of line is $\vec{r} = \langle 1, 1, 1 \rangle + t\langle 1, 2, -1 \rangle$

c) Write a definite integral for the arc length of the curve between $t = 1$ and $t = 2$ (do not evaluate the integral)

$s = \int_a^b \|\vec{v}\| dt = \int_1^2 \sqrt{1 + 4t^2 + t^{-4}}\, dt$

d) Sketch the curve (hint: Plot the point for $t = 1$, and consider where the curve is going as $t \to 0^+$ and as $t \to \infty$). Include in your sketch the unit tangent vector calculated in part b).



The projection in the xy plane is $\langle t, t^2 \rangle$

which is the parabola $y = x^2$

As $t \to 0^+$, $z \to \infty$

As $t \to \infty$, $z \to 0$ and the curve looks like $y = x^2$