Practice problems:

1) Find a basis of solutions of $A\bar{x} = \bar{0}$ if

$$A = \begin{bmatrix} 1 & 2 & 0 & 3 & 4 & 0 & 2 \\ 0 & 0 & 1 & 0 & -1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2) Determine if the vectors $\begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 2 \\ 4 \end{bmatrix}$ are linearly independent. When do you first know the answer to this question? If the vectors are linearly dependent, find a nontrivial linear combination that gives the zero vector, and express one of the vectors in terms of the others.

3) Consider the row equivalent matrices:

$$C = \begin{bmatrix} 1 & 2 & -1 & 3 & 5 & 0 & -2 \\ 2 & 4 & 0 & 6 & 8 & 1 & 9 \\ 1 & 2 & 2 & 3 & 2 & 1 & 15 \\ 1 & 2 & 1 & 3 & 3 & 1 & 11 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & 3 & 4 & 0 & 2 \\ 0 & 0 & 1 & 0 & -1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = R$$

Answer the following questions:

a) What is the general solution of $\begin{bmatrix} 1 & 2 & -1 & 3 & 5 & 0 \\ 2 & 4 & 0 & 6 & 8 & 1 \\ 1 & 2 & 2 & 3 & 2 & 1 \\ 1 & 2 & 1 & 3 & 3 & 1 \end{bmatrix} \bar{x} = \begin{bmatrix} -2 \\ 9 \\ 15 \\ 11 \end{bmatrix}$

b) What is the general solution of $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 0 & 6 \\ 1 & 2 & 2 & 3 \\ 1 & 2 & 1 & 3 \end{bmatrix} \bar{x} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ (note - this system comes from cols 1-4 of C as the matrix of coeffs and col 6 of C as the right hand side)

c) Express column 5 of C as a linear combination of columns 1 and 3 of C. Check your answer with the actual columns. Is this linear combination the only way to do it?

4) Using Gaussian elimination to put the augmented matrix into row reduced echelon form,

find the general solution of :

$$\begin{bmatrix} 1 & 2 & -1 & 3 & 5 & 0 \\ 2 & 4 & 0 & 6 & 8 & 1 \\ 1 & 2 & 2 & 3 & 2 & 1 \end{bmatrix} \bar{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

5) For each set of conditions below, give a nontrivial example of a row reduced echelon matrix R satisfying the condition. State what can be said concerning existence and uniqueness of solutions for linear systems $A\bar{x} = \bar{b}$ when using a matrix A that is row equivalent to your example R.

a) $r=m$, $r<n$

b) $r<m$, $r=n$

c) $r<m$, $r<n$

d) $r=m$, $r=n$