Math 251 Exam 2

1. Given the vector \( \mathbf{v} = \langle -1, 2, -3 \rangle \), use the dot product to determine which of the vectors \( \langle 2, 1, -2 \rangle \) or \( \langle 3, 1, -2 \rangle \) makes the smallest angle with the vector \( \mathbf{v} \). Show your work and explain your answer.

2. Calculate the projection of the vector \( \mathbf{u} = \langle 3, -1, 2 \rangle \) onto the vector \( \mathbf{v} = \langle 2, 1, 2 \rangle \) and show that the difference \( \mathbf{u} - \text{proj}_\mathbf{v} \mathbf{u} \) is orthogonal to \( \mathbf{v} \).

3. Find the equation of the line passing through the point \( (-1, 1, 2) \) in the direction of the vector \( \langle 2, 2, 1 \rangle \). Determine where this line intersects the plane \( x + 2y - z = 9 \).
4. Find the equation of the plane that contains the point $(2, 1, 1)$ and the line 
\[ \vec{r} = (2, -1, 2) + t(-1, 2, 3) \]

5. Find the equation of the plane that contains the point $(1, 1, -2)$ and has normal $(2, 2, 1)$. 
Find the distance of the point $(3, 2, 1)$ from this plane.
6. For the curve \( \vec{r}(t) = \langle t \cos t, t \sin t, e^{-t} \rangle, \ t \geq 0 \):
   
   a) Sketch and describe the curve.

   b) Write a definite integral for the arc length of the curve from \( t = 0 \) to \( t = 1 \). Simplify the integrand as appropriate, but do not evaluate the integral.

   c) Find a vector tangent to the curve when \( t = 0 \).

   d) Find the velocity, acceleration, and curvature, at \( t = 0 \).
7. Write equations for surfaces with the given descriptions:

   a) A right circular cylinder of radius 2 down the y-axis

   b) A "pill" in the shape of an ellipsoid with x and y intercepts equal to 2 and z intercept 1/2.

   c) A cone down the x axis whose cross-sections $x = k$ are twice as wide (in the y direction) as they are tall (in the z direction).

8. Sketch and classify:
   a) $-x^2 - y^2 + z^2 = 1$ Include the $x = 0$ cross section. What do the cross-sections for $z = k$ look like?

   b) $x = y^2 + \frac{z^2}{4}$ Include the cross section $x = 1$ and the cross section $z = 0$. 