Math 251 Exam 1

1. Find the general solution in vector form of the system

$$\begin{bmatrix} 1 & -1 & 0 & 1 & 2 \\ 2 & -2 & 0 & 0 & 4 \\ 1 & 1 & -1 & 1 & -1 \end{bmatrix} \bar{x} = \begin{bmatrix} 1 \\ -4 \\ 4 \end{bmatrix}$$

2. Evaluate $\det\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & x & -1 & 0 \\ 0 & 1 & x & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$

3. Let A be an $m \times n$ matrix. What can be said about solutions of the system $A\bar{x} = \bar{b}$ under the assumptions below. Briefly explain:

a) $rank\, A < m$

b) $rank\, A = n$

c) $m = n$ and $rank\, A = m$

d) $m = n$ and $A\bar{x} = \bar{0}$ has a nonzero solution

4. Evaluate the following determinant using a series of elimination steps and expansion (pivotal condensation) - reduce down to a 2x2 and then evaluate the 2x2 determinant.

$$\det\begin{bmatrix} -1 & 1 & 0 & 1 \\ 3 & 0 & 1 & 2 \\ 2 & 1 & 1 & 0 \\ 0 & 2 & -1 & 3 \end{bmatrix}$$

5. Determine whether the vectors $\begin{bmatrix} 1 \\ 3 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ -1 \\ 5 \end{bmatrix}$, are linearly independent. If they are linearly dependent, provide a nonzero linear combination that gives the zero vector.

6. If $A = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 0 & 1 \\ 2 & -1 & 2 \end{bmatrix}$

a) Calculate A^{-1} using row operations, and check your answer.

b) Use your answer in part a) to solve $A\bar{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

c) Check your value for $(A^{-1})_{23}$ by calculating the value of this element using cofactors.