Math 222 HW5 Due Wednesday, Dec. 2 before class (strict deadline!)

Note: In problems 1 and 2, do not use any functions we developed for interpolation, polynomial evaluation, or least squares; you should be able to do it the command window (or via a script file) using only built-in MATLAB commands.

1. Interpolate the function $f(x) = \frac{1}{1+x}$ on the inteval $[0,1]$ at the points $0, 0.2, 0.4, .., 1.$ On the same figure plot $f(x)$, $p(x)$ (the interpolant), and the datapoints. On a separate figure, plot $f(x) - p(x)$

2. Find a least-squares polynomial fit of the function $f(x) = \frac{1}{1+x}$ on the inteval $[0,1]$, obtained by sampling the function at the values $x = 0, .01, .02, .., .99, 1.$ Use the same degree polynomial as was used in problem 1. On the same figure, plot $f(x)$ and $p(x)$ (the least squares fit). On a separate figure, plot $f(x) - p(x)$. Which polynomial, the interpolant from problem 1 or the least squares approximation from problem 2, seems to give the better approximation?

3. Adapt the function cubeinterp.m as follows: Recall that we are given (x,y) data from a function and then, given a value $t,$ we approximate $f(t)$ with a value s obtained by interpolating the data (x,y) at the four datapoints nearest $t.$ What I'd like you to do is:
First, check whether t is represented among the datapoints x, and if it is, s should be the correpsonding y.
Also: if there is only one datapoint to the left of $t,$ i.e. x(1)<t<x(2), then you use the points x(1:4) to interpolate and if there is only one datapoint to the right of t, i.e. x(n-1)<t<x(n) then you use the points x((n-3):n) to interpolate.
Test your function out on an example, e.g. $f(x) = \sin x$ on $[0,\pi]$ where $x = 0 : \pi/20 : \pi$.

4. Plot the surface $z = f(x,y) = x^2 + 3xy + y^2$ for $-2 \leq x \leq 2$, $-2 \leq y \leq 2$.

5. Plot the surface $z = r^2 e^{-r}$ for $0 \leq r \leq 2$.