Introduction

Determining qualitative properties of solutions of dynamical systems arising from nonlinear interactions is generally a daunting task. A relevant mathematical theory that permits to biochemical interactions obeying mass-action laws is a triple \( (\mathbb{Z}^n, \mathbb{R}^n, \mathbb{R}^m) \), where \( \mathbb{Z}^n \) is the set of integer points, \( \mathbb{R}^n \) is the set of real vectors, and \( \mathbb{R}^m \) is the set of real numbers. The class of endotactic networks includes a simple geometric property. The class of endotactic networks is larger than the class of weakly reversible networks.

We prove the Persistence Conjecture for the case of networks with two-dimensional stoichiometric subspace. We illustrate the notion of endotactic networks for the case of two species in Figure 1. Let \( \mathbf{A} \) be a simple geometric property. The class of endotactic networks is larger than the class of weakly reversible networks.

Endotactic networks

Our results are applicable to endotactic networks, which is a large class of reaction networks characterized by a simple geometric property. The class of endotactic networks is larger than the class of weakly reversible networks.

The dynamics of weakly reversible population processes near facets, curves, and invariant subspaces is a major open question in the field of dynamical systems. We illustrate the notion of endotactic networks for the case of two species in Figure 1. Let \( \mathbf{A} \) be a simple geometric property. The class of endotactic networks is larger than the class of weakly reversible networks.

Examples

The classical two-species predator-prey model \( A + B \rightarrow 2A \rightarrow 2B \) is not endotactic, as one can see in Figure 7.

Two theorems

This fact is illustrated in Figure 5, where a trajectory of a fixed-parameter Lotka-Volterra system is depicted in black, and the trajectory of a variable Lotka-Volterra system with the same initial conditions is depicted using color.

The Global Attractor Conjecture is the central open problem in Chemical Reaction Network Theory. It is concerned with the global asymptotic stability of positive equilibria for the class of “complex-balanced” [5]. For the proof we use Theorem 2 to construct a hypersurface that separates a given trajectory from the boundary of \( \mathbb{R}_+^n \), as illustrated in Figure 8.

The Global Attractor Conjecture

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