

Compute the equations of a line in space

Example(1): Write parametric and symmetric equations of a line passing through $P(2, 3, -4)$ and parallel to $\mathbf{v} = (1, -1, 2)$.

Solution: The parametric and symmetric equations are, respectively,

$$\begin{cases} x = 2 + t \\ y = 3 - t \\ z = -4 + 2t \end{cases} \quad \text{and} \quad \frac{x-2}{1} = \frac{y-3}{-1} = \frac{z+4}{2}.$$

Example(2): Write parametric and symmetric equations of a line passing through $P(2, 5, -7)$ and $Q(4, 3, 8)$.

Solution: Note that the line is parallel to the vector $\mathbf{v} = \overline{PQ} = (2, -2, 15)$. Thus the parametric and symmetric equations are, respectively,

$$\begin{cases} x = 2 + 2t \\ y = 5 - 2t \\ z = -7 + 15t \end{cases} \quad \text{and} \quad \frac{x-2}{2} = \frac{y-5}{-2} = \frac{z+7}{15}.$$

Example(3): Write parametric and symmetric equations of a line passing through $P(2, 3, -4)$ and perpendicular to the plane $x + 2y + 3z = 4$.

Solution: Thus the line is parallel to a normal vector $\mathbf{n} = (1, 2, 3)$ of the plane, and so the parametric and symmetric equations are, respectively,

$$\begin{cases} x = 2 + t \\ y = 3 + 2t \\ z = -4 + 3t \end{cases} \quad \text{and} \quad \frac{x-2}{1} = \frac{y-3}{2} = \frac{z+4}{3}.$$