Write the equation of a sphere

Example: Write the equation of a sphere one of whose diameter is the line segment joining $(3, 5, -3)$ and $(7, 3, 1)$.

Solution: Find the center (x_0, y_0, z_0) of the sphere by computing the midpoint of the line segment.

$$x_0 = \frac{3+7}{2} = 5, y_0 = \frac{5+3}{2} = 4, z_0 = \frac{-3+1}{2} = -1.$$

The radius of the sphere is the distance from the center to one of the points (say $(3, 5, -3)$).

$$r^2 = (5-3)^2 + (4-5)^2 + (-1-(-3))^2 = 4 + 1 + 4 = 9.$$

Answer: The equation of the sphere is

$$(x-5)^2 + (y-4)^2 + (z+1)^2 = 9.$$

Compute the center and the radius of a sphere

Example: Find the center and the radius of the following sphere,

$$x^2 + y^2 + z^2 - 8x - 6y + 10z + 34 = 0.$$

[5pt] **Solution**: Completing the squares to get

$$x^2 - 8x = (x-4)^2 - 16, y^2 - 6y = (y-3)^2 - 9, z^2 + 10z = (z+5)^2 - 25,$$

and so $x^2 + y^2 + z^2 - 8x - 6y + 10z + 34 = 0$ is equivalent to $(x-4)^2 + (y-3)^2 + (z+5)^2 = 16$. Thus the center of the sphere is $(4, 3, -5)$ and the radius is 4.

Determine if two vectors are parallel or perpendicular

Example(1): Suppose $\mathbf{a} = (12, -20, 16)$ and $\mathbf{b} = (-9, 15, -12)$. Note that $\mathbf{a} = \frac{4}{3}\mathbf{b}$. Thus $\mathbf{a}$ and $\mathbf{b}$ are parallel.

Example(2): Suppose $\mathbf{a} = (12, -20, 16)$ and $\mathbf{b} = (-9, 15, 24)$. From Example (1) we know that the two vectors are not parallel. Note that $\mathbf{a} \cdot \mathbf{b} = (12)(-9) + (-20)(15) + (16)(24) = -108 - 300 + 384 \neq 0$. Thus $\mathbf{a}$ and $\mathbf{b}$ are neither parallel nor perpendicular.