Determine the largest possible domain of a function

Example (1) : Determine the largest possible domain of $f(x,y) = (\sqrt{2x} + \sqrt[3]{3y}$.

Solution: Any real value of y can make $\sqrt[3]{3y}$ meaningful, and so the domain for $\sqrt[3]{3y}$ is the whole y-axis. Only non negative real value of x can make $\sqrt{2x}$ meaningful, and so the domain for $\sqrt{2x}$ is the half line $[0,\infty)$. Combining these facts, we conclude that the domain of the function $f(x,y) = (\sqrt{2x} + \sqrt[3]{3y}$ is the half plane where $x \geq 0$, or in set notation: $\{(x,y) : 0 \leq x < \infty$ and $-\infty < y < \infty\}$.

Example (2) : Determine the largest possible domain of $f(x,y) = \dfrac{xy}{x^2 - y^2}$.

Solution: To avoid zero denominators, we must have $x^2 - y^2 \neq 0$. Since $x^2 - y^2 = (x-y)(x+y)$, the domain of this function is the whole xy-plane with the two straight lines $y = x$ and $y = -x$ taken away.