Convert equations from one coordinate system to another: II

Useful Facts

Cylindrical	Rectangle	Spherical	Rectangle
$\begin{cases} r^2 &= x^2+y^2 \\ \theta &= \tan^{-1}\frac{y}{x} \\ z &= z \end{cases}$	$\begin{cases} x &= r\cos\theta \\ y &= r\sin\theta \\ z &= z \end{cases}$	$\begin{cases} \rho^2 &= x^2+y^2+z^2 \\ \phi &= \tan^{-1}\frac{\sqrt{x^2+y^2}}{z} \\ \theta &= \tan^{-1}\frac{y}{x} \end{cases}$	$\begin{cases} x &= \rho\sin\phi\cos\theta \\ y &= \rho\sin\phi\sin\theta \\ z &= \rho\cos\phi \end{cases}$

Example (4) : Convert the equation $x^2+y^2 = 2x$ to both cylindrical and spherical coordinates.

Solution: Apply the Useful Facts above to get (for cylindrical coordinates)

$$r^2 = 2r\cos\theta, \text{ or simply } r = 2\cos\theta;$$

and (for spherical coordinates)

$$\rho^2\sin^2\phi = 2\rho\sin\phi\cos\theta \text{ or simply } \rho\sin\phi = 2\cos\theta.$$

Example (5) : Describe the graph $r = 4\cos\theta$ in cylindrical coordinates.

Solution: Multiplying both sides by r to get $r^2 = 4r\cos\theta$. Then apply the Useful Facts to get $x^2+y^2 = 4x$. Completing the squares, we obtain $(x-2)^2+y^2 = 4$, and so this is a vertical cylinder whose axis is the straight line $L : x = 2, y = 0, z = t$.

Example (6) : Describe the graph $\rho = 4\cos\phi$ in spherical coordinates.

Solution: Multiplying both sides by ρ to get $\rho^2 = 4\rho\cos\theta$. Then apply the Useful Facts to get $x^2+y^2+z^2 = 4z$. Completing the squares, we obtain $x^2+y^2+(z-2)^2 = 4$, and so this is a sphere whose center is at $(0,0,2)$ with radius 2.

Example (7) : Convert the equation $z = x^2-y^2$ to both cylindrical and spherical coordinates.

Solution: Apply the Useful Facts to get (for cylindrical coordinates)

$$z = r^2(\cos^2\theta - \sin^2\theta),$$

and (for spherical coordinates)

$$\rho\cos\theta = \rho^2\sin^2\phi(\cos^2\theta-\sin^2\theta), \text{ or simply } \cos\theta = \rho\sin^2\phi(\cos^2\theta-\sin^2\theta).$$