Operations involving 3D vectors

Example: Given $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ and $\mathbf{b} = 5\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$, compute $2\mathbf{a} + \mathbf{b}$, $3\mathbf{a} - 2\mathbf{b}$, $\mathbf{a} \cdot \mathbf{b}$, $|\mathbf{a} - \mathbf{b}|$, and $\mathbf{a}/|\mathbf{a}|$.

Solution: Then $\mathbf{a} = (2, -3, 5)$ and $\mathbf{b} = (5, 3, -7)$.

$$\begin{aligned}
2\mathbf{a} + \mathbf{b} &= (4, -6, 10) + (5, 3, -7) = (9, -3, 3). \\
3\mathbf{a} - 2\mathbf{b} &= (6, -9, 15) - (10, 6, -14) = (-4, -15, 29). \\
\mathbf{a} \cdot \mathbf{b} &= (2)(5) + (-3)(3) + (5)(-7) = 10 - 9 - 35 = -34. \\
|\mathbf{a} - \mathbf{b}| &= |(2 - 5, -3 - 3, 5 - (-7))| = \sqrt{9 + 36 + 144} = \sqrt{189}. \\
\mathbf{a}/|\mathbf{a}| &= (2, -3, 5)/\sqrt{4 + 9 + 25} = \left(\frac{4}{\sqrt{38}}, \frac{-3}{\sqrt{38}}, \frac{5}{\sqrt{38}}\right).
\end{aligned}$$

Compute the angle between two vectors

Example: Given $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ and $\mathbf{b} = 5\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$, compute the angle between $\mathbf{a}$ and $\mathbf{b}$.

Solution: Compute $|\mathbf{b}| = \sqrt{25 + 9 + 49} = \sqrt{83}$. From $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| \cdot |\mathbf{b}| \cos\theta$, we have (utilizing the answers for $\mathbf{a} \cdot \mathbf{b} = -34$ and $|\mathbf{a}| = \sqrt{38}$ in the example above),

$$\cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| \cdot |\mathbf{b}|} = \frac{-34}{\sqrt{38}\sqrt{83}} = \frac{-34}{\sqrt{249}}; \text{ and so } \theta = \cos^{-1}\frac{-34}{\sqrt{249}}.$$

Compute component of a along b

Example: Given $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ and $\mathbf{b} = 5\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$, compute $\text{Comp}_{\mathbf{a}}\mathbf{b}$ and $\text{Comp}_{\mathbf{b}}\mathbf{a}$.

Solution: Recall that

$$\text{Comp}_{\mathbf{b}}\mathbf{a} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|}.$$

We compute $\mathbf{a} \cdots \mathbf{b} = -34$, $|\mathbf{a}| = \sqrt{38}$ and $|\mathbf{b}| = \sqrt{83}$ (see examples above), and so

$$\text{Comp}_{\mathbf{b}}\mathbf{a} = \frac{-34}{\sqrt{83}} \text{ and } \text{Comp}_{\mathbf{a}}\mathbf{b} = \frac{-34}{\sqrt{38}}.$$