Determine the projection of the intersection curve of two surfaces

Example (1): Prove the projection into the yz-plane of the curve of intersection of the surfaces $x = 1 - y^2$ and $x = y^2 + z^2$ is an ellipse.

Solution: We are to determine the equation of such a projection. As the projection is on the yz-plane, this equation should be one without the variable x. To find the equation of the projection, we cancel x in the system of equations (obtained by combining the equations of the two surfaces)

$$\begin{cases} x = 1 - y^2 \\ x = y^2 + z^2 \end{cases} \tag{1}$$

to get $1 - y^2 = y^2 + z^2$. With some algebra, we can rewrite this equation into

$$2y^2 + z^2 = 1,$$

which is an ellipse on the yz-plane.

Example (2): Prove the projection into the xz-plane of the curve of intersection of the paraboloids $y = 2x^2 + 3z^2$ and $y = 5 - 3x^2 - 2z^2$ is a circle.

Solution: We are to determine the equation of such a projection. As the projection is on the xz-plane, this equation should be one without the variable y. To find the equation of the projection, we cancel y in the system of equations (obtained by combining the equations of the two surfaces)

$$\begin{cases} y = 2x^2 + 3z^2 \\ y = 5 - 3x^2 - 2z^2 \end{cases} \tag{2}$$

to get $2x^2 + 3z^2 = 5 - 3x^2 - 2z^2$. With some algebra, we can rewrite this equation into

$$x^2 + z^2 = 1,$$

which is a circle on the xz-plane.