Write equations of a line as intersections of two planes

Example: Write the parametric and sysmetric equations of the line of intersection of the planes $2x - y + z = 5$ and $x + y - z = 1$.

Solution: The planes have normal vectors $\mathbf{a} = (2, -1, 1)$ and $\mathbf{b} = (1, 1, -1)$, respectively. Let L denote the line of intersection. Then $\mathbf{v} = \mathbf{a} \times \mathbf{b} = (1 - 1, -(-2 - 1), 2 - (-1)) = (0, 3, 3)$ is parallel to L. We only need to find a point P on L.

To find P, solved the system of equations of the planes:

$$2x - y + z = 5 \text{ and } x + y - z = 1.$$

We consider P to be the point of L on the plane $z = 0$. Thus substitute $z = 0$ in the ssytem above to get

$$2x - y = 5 \text{ and } x + y = 1 \Longrightarrow x = 2, y = -1.$$

Hence we get $P(2, -1, 0)$, and so the equations of the line are

$$\begin{cases} x = 2 \\ y = -1 + 3t \\ z = 3t \end{cases} \quad \text{and } x = 2, y + 1 = z.$$