Determine the equation of a straight line in the plane

Some Facts about lines

(1) For non vertical line L, if L passes a point (x_0, y_0) with slope m, then an equation of line L is

$$y - y_0 = m(x - x_0).$$

(2) If a non vertical line L passes through distinct points (x_1, y_1) and $x_2(y_2)$, then the slope of L is

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

(3) A vertical line passing through (x_0, y_0) has equation $x = x_0$; a horizontal line passing through (x_0, y_0) has equation $y = y_0$.

(4) A line L intersects the x-axis at $(a, 0)$ (the number a will be called the x-intersect of L) and the y-axis at $(0, b)$ (the number b will be called the y-intersect of L) has equation

$$\frac{x}{a} + \frac{y}{b} = 1.$$

(5) Two non vertical lines are parallel if and only if they have the same slope; and two non vertical lines are perpendicular to each other if and only if the product of their slopes equals -1.

(6) The angle θ between the x-axis and a non vertical line L is called an **angle of inclination**, and the slope of L is $m = \tan\theta$.

Example 1 Find an equation of the line L that passes through $(2, -3)$ and $(5, 3)$.

Solution: Then the slope of L is

$$m = \frac{3 - (-3)}{5 - 2} = \frac{6}{3} = 2.$$

As L passes through $(2, -3)$, an equation of L is

$$y - (-3) = 2(x - 2).$$

Example 2 Find an equation of the line L that has slope 6 and y-intersect 7.

Solution: The line has y-intersect 7 means that $(0, 7)$ is on the line. Therefore an equation of L is

$$y - 7 = 6(x - 0).$$

Example 3 Find an equation of the line L that has angle of inclination 135° and contains $(4, 2)$.

Solution: The slope of L is $m = \tan 135^o = \tan \frac{3\pi}{4} = -1$ Therefore an equation of L is

$$y - 2 = -1(x - 4).$$

Example 4 Find an equation of the line L that passes through $(1, 5)$ and is parallel to the line with equation $2x + y = 10$.

Solution: Rewrite $2x + y = 10$ into $y = -2x + 10$. Then the slope of this line is -2. Note that L also has the same slope and so an equation of L is

$$y - 5 = -2(x - 1).$$

Example 5 Find an equation of the line L that passes through $(1, 5)$ and is perpendicular to the line with equation $2x + y = 10$.

Solution: Rewrite $2x + y = 10$ into $y = -2x + 10$. Then the slope of this line is -2. Let m denote the slope of L. Then $(-2)m = -1$, and so $m = \frac{1}{2}$. Thus an equation of L is

$$y - 5 = \frac{1}{2}(x - 1).$$