Find derivatives by using differentiation rules

Differentiation Rules:

(1) Derivative of a constant: Let C be a constant, then $\frac{d}{dx}C = 0$.

(2) Power Rule: For a real number n,

$$\frac{dx^n}{dx} = nx^{n-1}.$$

(3) Linear Property: For constant a and b and functions $f(x)$ and $g(x)$,

$$[af(x) + bg(x)]' = af'(x) + bg'(x) \text{ and } [af(x) - bg(x)]' = af'(x) - bg'(x).$$

(4) Product Rule: $[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$.

(5) Quotient Rule:

$$\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}.$$

(6) Generalized Power Rule: for a real number n and a differentiable function $f(x)$,

$$\frac{d}{dx}[f(x)]^n = n[f(x)]^{n-1}f'(x).$$

Example 1 Apply differentiation rules to find the derivative of $f(x) = (2x^2 - 1)(x^3 + 2)$.

Solution: The function $f(x)$ is a product, and each factor is a polynomial. So we first apply Product Rule, and then apply the linear property and the power rule to get:

$$\begin{aligned} f'(x) &= \underline{(2(2)x^{2-1} - 0)}(x^3 + 2) + (2x^2 - 1)\underline{(3x^{3-1} + 0)} = 4x(x^3 + 2) + 3x^2(2x^2 - 1) \\ &= 4x^4 + 8x + 6x^4 - 3x^2 = 10x^4 - 3x^2 + 8x. \end{aligned}$$

Example 2 Apply differentiation rules to find the derivative of $f(x) = \frac{2x^2 - 1}{x^3 + 2}$.

Solution 1: The function $f(x)$ is a quotient. So we first apply Quotient Rule, and then apply the linear property and the generalized power rule to get:

$$\begin{aligned} f'(x) &= \frac{\underline{(2(2)x^{2-1} - 0)}(x^3 + 2) - (2x^2 - 1)\underline{(3x^{3-1} + 0)}}{(x^3 + 2)^2} = \frac{4x(x^3 + 2) + 3x^2(2x^2 - 1)}{(x^3 + 2)^2} \\ &= \frac{4x^4 + 8x + 6x^4 - 3x^2}{(x^3 + 2)^2} = \frac{x(10x^3 - 3x + 8)}{(x^3 + 2)^2}. \end{aligned}$$

Solution 2: View the function $f(x)$ as a product by using negative exponents.

$$f(x) = (2x^2 - 1)(x^3 + 2)^{-1}.$$

Then apply Product Rule, and then the linear property and the power rule to get the answer (the answer is intentionally not simplified to make it easier for a reader to see the computation process).

$$\begin{aligned} f'(x) &= \underline{(2(2)x^{2-1}-0)}(x^3+2)^{-1}+(2x^2-1)\underline{(-1)(x^3+2)^{-1-1}(3x^{3-1}+0)} \\ &= 4x(x^3+2)^{-1}-3x^2(2x^2-1)(x^3+2)^{-2}. \end{aligned}$$

Example 3 Apply differentiation rules to find the derivative of $f(x) = \dfrac{2x^3-3x^2+4x-5}{x^2}$.

Solution: The function $f(x)$ is a quotient. But we are not in a hurry to apply Quotient Rule, as we observed that the denominator is just a power of x. Thus we first simplify the fraction.

$$f(x) = \frac{2x^3-3x^2+4x-5}{x^2} = \frac{2x^3}{x^2} - \frac{3x^2}{x^2} + \frac{4x}{x^2} - \frac{5}{x^2} = 2x-3+4x^{-1}-5x^{-2}.$$

Then apply the linear property and the power rule.

$$f'(x) = [2x-3+4x^{-1}-5x^{-2}]' = 2-0+4(-1)x^{-1-1}-5(-2)x^{-2-1} = 2-\frac{4}{x^2}+\frac{10}{x^3}.$$

Example 4 Write an equation of the line tangent to the curve $f(x) = \left(\dfrac{2}{x}-\dfrac{1}{x^2}\right)^{-1}$ at the point $(2, 4/3)$.

Solution: The equation of this line has the form

$$y - \underline{\frac{4}{3}} = \underline{f'(2)}(x-\underline{2}).$$

To find the slope $f'(2)$, we first compute the derivative $f'(x)$. To do that it may be better to simplify the fraction first

$$f(x) = \left(\frac{2}{x}-\frac{1}{x^2}\right)^{-1} = \left(\frac{2x}{x^2}-\frac{1}{x^2}\right)^{-1} = \left(\frac{2x-1}{x^2}\right)^{-1} = \frac{x^2}{2x-1}.$$

Then, apply Quotient Rule

$$f'(x) = \frac{\underline{2x}(2x-1)-\underline{2}x^2}{(2x-1)^2} = \frac{4x^2-2x-2x^2}{(2x-1)^2} = \frac{2x^2-2x}{(2x-1)^2}.$$

Hence $f'(2) = \frac{4}{9}$, and so the equation is

$$y - \underline{\frac{4}{3}} = \underline{\frac{4}{9}}(x-\underline{2}).$$