

Application of the intermediate value property of continuous functions

Intermediate Value Property:

If a function $f(x)$ is continuous on a closed interval $[a, b]$, and if K is a number between $f(a)$ and $f(b)$, then there must be a point c in the interval $[a, b]$ such that $f(c) = K$.

This property is often used to show the existence of an equation.

Example 1 Apply intermediate value property to show that the equation $x^5 - 3x^2 = -1$ has a solution in the interval $[0, 1]$.

Solution: Let $f(x) = x^5 - 3x^2$. Then $f(x)$ is a continuous function with $f(0) = 0$ and $f(1) = -2$. As -1 is a value between -2 and 0 , the intermediate value property of continuous functions indicates that $f(x) = -1$ must have a solution in the interval $[0, 1]$.

Example 2 Apply intermediate value property to show that the equation $x^5 - 5x^3 + 3 = 0$ has a solution in the interval $[-1, 1]$.

Solution: Let $f(x) = x^5 - 5x^3 + 3$. Then $f(x)$ is a continuous function with $f(-1) = 7$ and $f(1) = -1$. As 0 is a value between -1 and 7 , the intermediate value property of continuous functions indicates that $f(x) = 0$ must have a solution in the interval $[-1, 1]$.

Example 3 Apply intermediate value property to show that the equation $\sqrt{x^6 + 5x^4 + 9} = 3.5$ has a solution in the interval $[0, 1]$.

Solution: Let $f(x) = \sqrt{x^6 + 5x^4 + 9}$. Then $f(x)$ is a continuous function with $f(0) = 3$ and $f(1) = 4$. As 3.5 is a value between 3 and 4 , the intermediate value property of continuous functions indicates that $f(x) = 3.5$ must have a solution in the interval $[0, 1]$.