MATH 251 - QUIZ 6

NAME:

I.D.:

Instruction: Circle your answers and show all your work CLEARLY. Solutions with answer only and without supporting procedures will have little credit.

1. Find and classify the critical point of the function $f(x, y) = x^2 - 2xy + y^3 - y$.

Solution: Compute the partial derivatives:

$$f_x = 2x - 2y \text{ and } f_y = -2x + 3y^2 - 1.$$

Set $f_x = 0$ and $f_y = 0$. From $f_x = 0$, we have $x = y$. Substitute $x = y$ in the equation $f_y = 0$ to get $3y^2 - 2y + 1 = 0$, which is the same as $(3y + 1)(y - 1) = 0$. Therefore, $y = -\frac{1}{3}$ or $y = 1$. As $x = y$, the critical points are $(\frac{1}{3}, \frac{1}{3})$ and $(1, 1)$. Note that $f(-\frac{1}{3}, -\frac{1}{3}) = \frac{5}{27}$ and $f(1, 1) = -1$.

Next, compute the second order of derivatives.

$$f_{xx} = 2, f_{yy} = 6y - 2 \text{ and } f_{xy} = -2.$$

At $(-\frac{1}{3}, -\frac{1}{3})$, $\Delta = 0 - 4 < 0$, and so the surface has a saddle point at $(-\frac{1}{3}, -\frac{1}{3}\frac{5}{27})$. At $(1, 1)$, $\Delta = 8 > 0$ and $f_{xx} > 0$. Thus the surface has a local minimum at $(1, 1, -1)$.

2. Find and classify the critical point of the function $f(x, y) = x^3 + y^3 + 3xy + 3$.

Solution: Compute $f_x = 3x^2 + 3y$ and $f_y = 3y^2 + 3x$. Solve $f_x = 3x^2 + 3y = 0$ and $f_y = 3y^2 + 3x = 0$. Note that both $x \le 0$ and $y \le 0$. Combine the two equations to get $x(x^3 + 1) = 0$ and so $x = 0$ or $x = -1$. As $y \le 0$, we substitute $x = 0$ and $x = -1$ respectively to get two critical points $(0, 0)$ and $(-1, -1)$. Note that $f(0, 0) = 3$ and $f(-1, -1) = 4$.

Compute $f_{xx} = 6x, f_{xy} = 3$ and $f_{yy} = 6y$. As $\Delta(0, 0) = -9 < 0$, the surface has a saddle point at $(0, 0, 3)$. As $\Delta(-1, -1) = 27 > 0$ and $f_{xx}(-1, -1) = -6 < 0$, the surface has a local maximum at $(-1, -1, 4)$.