MATH 251 - QUIZ 4

NAME:

I.D.:

Instruction: Circle your answers and show all your work CLEARLY. Solutions with answer only and without supporting procedures will have little credit.

1. Use limit laws to compute the following limits, or indicate that the limit does not exist.

(1A) $\displaystyle\lim_{(x,y)\to(-1,-1)} \frac{5-x^2}{3+x+y}$.

Solution As the function $\frac{5-x^2}{3+x+y}$ is continuous at $(-1,-1)$, (that is, $(-1,-1)$ is in the domain of the fractional function $\frac{5-x^2}{3+x+y}$), taking the limit is the same as evaluating the function:

$$\lim_{(x,y)\to(-1,-1)} \frac{5-x^2}{3+x+y} = \frac{5-(-1)^2}{3+(-1)+(-1)} = \frac{4}{1} = 4.$$

(1B) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\sin(xy)}{xy}$.

Solution Recall that in Calculus I, we have learned a basic limit $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$ (this basic limit can also be obtained by L'Hôpital's rule in Calculus II). Compare with the current problem, we can set $\theta = xy$ to get

$$\lim_{(x,y)\to(0,0)} \frac{\sin(xy)}{xy} = \lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1.$$

Remark: Directly apply L'Hôpital's rule to this problem is erroneous, for it does satisfy the condition of the rule. Also, writing $\frac{\sin 0}{0} = 1$ is also wrong, as this expression, with 0 in the denominator, is meaningless.

(1C) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2}$.

Solution 1 Considering the limit is taken along the line $y = x$, we obtain the limit $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2} = \lim_{x\to 0} \frac{2x^2}{x^2+x^2} = 1$; and considering the limit is taken along the line $y = -x$, we obtain the limit $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2} = \lim_{x\to 0} \frac{-2x^2}{x^2+x^2} = -1$. Therefore, the limit does not exist.

Solution 2 Another way to do it is to set $x = r\cos\theta$ and $y = r\sin\theta$. Note that $r \to 0$ but θ does not go to zero. Instead, θ can take any value. The the limit, after cancelling r^2, becomes $2\sin\theta\cos\theta$. Therefore, when $\theta = 0$, the limit is 0; and when $\theta = \frac{\pi}{4}$, then limit is 1. Therefore, the limit does not exist.