Review for Chapter 17

1. A vector field is a vector function $\mathbf{F} =< P(x,y,z), Q(x,y,z), R(x,y,z) >$ or $\mathbf{F} = < P(x,y), Q(x,y) >$.

2. The gradiant operator $\nabla =< \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} >$ is a linear operator.

3. div $\mathbf{F} = \nabla \bullet \mathbf{F}$.

4. curl $\mathbf{F} = \nabla \times \mathbf{F}$.

5. Line integrals: If C is a curve with parametric equations $< x(t), y(t), z(t) >$ with $a \leq t \leq b$, then

$$\int_C f(x,y,z)dx = \int_a^b f(x(t), y(t), z(t))x'(t)dt.$$

$$\int_C f(x,y,z)dy = \int_a^b f(x(t), y(t), z(t))y'(t)dt.$$

$$\int_C f(x,y,z)dz = \int_a^b f(x(t), y(t), z(t))z'(t)dt.$$

$$\int_C f(x,y,z)ds = \int_a^b f(x(t), y(t), z(t))\sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}dt.$$

6. Convervative fields: A vector field $\mathbf{F} = < P, Q >$ is conservative if and only if $P_y = Q_x$. In this case, one can find its potential function f such that $\nabla f = \mathbf{F}$ by the following steps: (1) Find $f(x,y) = \int P dx + \phi(y)$.
(2) Set $Q = f_y$ to get a differential equation of $\phi'(y)$.
(3) Solve the differential equation to find $\phi(y)$ and thereby getting f.

7. Important fact: If $\mathbf{F} = < P, Q, R >$ is a conservative field, and if $\mathbf{T}$ is the unit tangent vector of the curve C, then $\int_C \mathbf{F} \bullet \mathbf{T} ds$ is independent of path. (Therefore, you can make use of it to simplify the integral).

8. Green's Theorem: Let C be the closed curve bounding the region R and suppose that P, Q are both continuous and have continuous first order of derivatives, then

$$\oint_C P dx + Q dy = \int\int_R (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}) dA.$$