Review for Exam 3

1. How to find tangent planes at a point $P(a,b,c)$? (For a surface with equation $z=f(x,y)$, use normal vector $\vec{n}(a,b) =< f_x(a,b), f_y(a,b), -1 >$. For surface with equation $F(x,y,z)=0$, the gradient $\nabla F(a,b,c)$ is a normal.

2. How to classify the critical points (if they are local extrema and what kind)? (Use $\triangle(x,y) = f_{xx}f_{yy} - (f_{xy})^2$.)

3. Differentials and its applications.

$$df(x,y,z) = \nabla f(x,y,z) \bullet < dx, dy, dz > .$$

One can use df as an approximation to $f(x+dx, y+dy, z+dz) - f(x,y,z)$.

4. Chain rules: (Go check them on pages 736 and 739).

5. Implicit partial differentiations. An equation $F(x,y,z)$ = constant, defines one variable (z, say) as a function of the other variables (x, y, say). Then

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \ \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}.$$

6. The gradient and the directional derivatives:

$$\nabla f(x,y,z) =< f_x, f_y, f_z > \text{ and } D_{\vec{n}} f(x,y,z) = \nabla f(x,y,z) \bullet \vec{n}, \text{ if } |\vec{n}| = 1.$$

7. Evaluation of double integrals: (x, y coordinates)

$$\int\int_R f(x,y)dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)dydx, \text{ if } R \text{ is } a \le x \le b,\ g_1(x) \le y \le g_2(x).$$

$$\int\int_R f(x,y)dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)dydx, \text{ if } R \text{ is } c \le y \le d,\ h_1(y) \le x \le h_2(y).$$

When you can evaluate the integral by either way, you may want to choose a simpler way.

8. Evaluation of double integrals: (Polar coordinates)

$$\int\int_R f(x,y)dA = \int_a^b \int_{g_1(r)}^{g_2(r)} f(r\cos\theta, r\sin\theta) r d\theta dr, \text{ if } R \text{ is } a \le r \le b,\ g_1(r) \le \theta \le g_2(r).$$

$$\int\int_R f(x,y)dA = \int_a^b \int_{h_1(\theta)}^{h_2(\theta)} f(r\cos\theta, r\sin\theta) r dr d\theta, \text{ if } R \text{ is } a \le \theta \le b,\ h_1(\theta) \le r \le h_2(\theta).$$

A useful fact: $dA = rdrd\theta = dxdy$.

9. Some applications of double integrals.

9a. Area of region R is $\int\int_R dA$.

9b. The volume beween $z = f(x,y)$ and $z = g(x,y)$ when (x,y) are in R is $\int\int_R (f(x,y) - g(x,y))dA$.