

## 1. Convergence and Divergence Tests for Series

Test	When to Use	Conclusions
Divergence Test	for any series $\sum_{n=0}^{\infty} a_n$	Diverges if $\lim_{n \rightarrow \infty}  a_n  \neq 0$ .
Integral Test	$\sum_{n=0}^{\infty} a_n$ with $a_n \geq 0$ and $a_n$ decreasing	$\int_1^{\infty} f(x)dx$ and $\sum_{n=0}^{\infty} a_n$ both converge/diverge where $f(n) = a_n$ .
Comparison Test	$\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ if $0 \leq a_n \leq b_n$	$\sum_{n=0}^{\infty} b_n$ converges $\implies \sum_{n=0}^{\infty} a_n$ converges. $\sum_{n=0}^{\infty} a_n$ diverges $\implies \sum_{n=0}^{\infty} b_n$ diverges.
Limiting Comparison Test	$\sum_{n=0}^{\infty} a_n, (a_n > 0)$ . Choose $\sum_{n=0}^{\infty} b_n, (b_n > 0)$ if $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L$ with $0 < L < \infty$	$\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ both converge/diverge
	if $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 0$	$\sum_{n=0}^{\infty} b_n$ converges $\implies \sum_{n=0}^{\infty} a_n$ converges.
	if $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \infty$	$\sum_{n=0}^{\infty} b_n$ diverges $\implies \sum_{n=0}^{\infty} a_n$ diverges.
Convergent test for alternating Series	$\sum_{n=0}^{\infty} (-1)^n a_n \quad (a_n > 0)$	converges if $\lim_{n \rightarrow \infty} a_n = 0$ and $a_n$ is decreasing
Absolute Convergence	for any series $\sum_{n=0}^{\infty} a_n$	If $\sum_{n=0}^{\infty}  a_n $ converges, then $\sum_{n=0}^{\infty} a_n$ converges, (definition of absolutely convergent series.)
Conditional Convergence	for any series $\sum_{n=0}^{\infty} a_n$	if $\sum_{n=0}^{\infty}  a_n $ diverges but $\sum_{n=0}^{\infty} a_n$ converges. $\sum_{n=0}^{\infty} a_n$ conditionally converges
Ratio Test:  Root Test:	For any series $\sum_{n=0}^{\infty} a_n$ , Calculate $\lim_{n \rightarrow \infty} \left  \frac{a_{n+1}}{a_n} \right  = L$  Calculate $\lim_{n \rightarrow \infty} \sqrt[n]{ a_n } = L$	there are 3 cases: if $L < 1$ , then $\sum_{n=0}^{\infty}  a_n $ converges ; if $L > 1$ , then $\sum_{n=0}^{\infty}  a_n $ diverges; if $L = 1$ , no conclusion can be made.

## 2. Important Series to Remember

Series	How do they look	Conclusions
Geometric Series	$\sum_{n=0}^{\infty} ar^n$	Converges to $\frac{a}{1-r}$ if $ r  < 1$ and diverges if $ r  \geq 1$
$p$ -series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	Converges if $p > 1$ and diverges if $p \leq 1$