**Math 156 Fall 2015 Quiz 6 Solutions**

**Name:**

**1**: Determine if the given sequence $\{a_n\}$ is convergent or divergent, and find the limit if convergent.

$$a_n = \frac{\ln(n^2)}{n}.$$

[5pt] **Solution** As this is a sequence, we directly compute the limit by using L'Hospital Rule

$$\lim_{n\to\infty} \frac{\ln(n^2)}{n} = \lim_{n\to\infty} \frac{\frac{2n}{n^2}}{1} = \lim_{n\to\infty} \frac{2}{n} = 0.$$

Hence the sequence is convergent and the limit is 0.

If we notice that $\ln(n^2) = 2\ln(n)$, then the computation can be simpler:

$$\lim_{n\to\infty} \frac{\ln(n^2)}{n} = \lim_{n\to\infty} \frac{2\ln(n)}{n} = \lim_{n\to\infty} \frac{2}{n} = 0.$$

**2**: Determine if the series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2n+1}$ is convergent or divergent. If it is convergent, find its sum.

**Solution** We do the routine by computing the limit of the generic term

$$\lim_{n\to\infty} \frac{n}{2n+1} = \frac{1}{2} \neq 0.$$

As the limit is not zero, the series diverges.

**3**: Express the repeating decimal $0.\overline{12} = 0.12121212...$ as a ratio of integers.

**Solution** First let us understand what w are doing:

$$\begin{aligned} 0.\overline{12} &= 0.12121212... = 0.12 + 0.0012 + 0.000012 + ... \\ &= 12\left(\frac{1}{100} + \frac{1}{100^2} + \frac{1}{100^3} + ...\right) = \sum_{n=1}^{\infty} \frac{12}{100^n}. \end{aligned}$$

We now recognize that this is a geometric series with $a = 12$ and $r = \frac{1}{100}$, starting from $n = 1$. Using the geometric summation formula, we compute the sum by firstly convert it into one that starts from $n = 0$.

$$0.\overline{12} = \sum_{n=1}^{\infty} \frac{12}{100^n} = \sum_{n=0}^{\infty} \frac{12}{100^n} - 12 = \frac{12}{1-\frac{1}{100}} - 12 = 12\left(\frac{100}{99} - 1\right) = \frac{12}{99} = \frac{4}{33}.$$

**Discussion:** This solution requires a small "surgery" on the series to convert it from starting with $n = 0$, so that we can apply the formula. This seems to be a bit annoying for some of the students. Two of us, Caitlin and Ornella, have found an easy way to do it without the surgery. Their way is to take out the first term 0.12 as a common factor instead of using 12 as a common factor above. In this way, the series will always starts with 1 (see below) and so the formula is applicable without a surgery. Here is Caitlin and Ornelle's solution (I consider it better than the one above): First write

$$\begin{aligned} 0.\overline{12} &= 0.12121212... = 0.12 + 0.0012 + 0.000012 + ... \\ &= 0.12\left(1 + \frac{1}{100} + \frac{1}{100^2} + \frac{1}{100^3} + ...\right) = \sum_{n=0}^{\infty} \frac{0.12}{100^n}. \end{aligned}$$

Hence $a = 0.12$ and $r = \frac{1}{100}$. Apply the formula, we now have

$$0.\overline{12} = \sum_{n=0}^{\infty} \frac{0.12}{100^n} == \frac{0.12}{1 - \frac{1}{100}} = \frac{0.12 \cdot 100}{99} = \frac{12}{99} = \frac{4}{33}.$$

**Grade Distribution and Discussions**:

**Meaning of the scores:**
**9, 10 = Very good**, familiar with the related materials and skillful, with minimal computational errors. Keep on!
**8 = good**, familiar with most of the related materials, with a few computations errors. Make an effort to do better.
**7 = Passing**, lack of familiarity of the related materials and more computational errors and algebraic errors. Set our goal as "familiar with the skills" instead of just knowing it.
**6 = Borderline**. We need to catch it up. If you have trouble doing your homework, it might be time to visit your instructor to get help. Do not wait to let the trouble accumulate.
**At most 5** = This might be a dangerous warning signal. **We are failing!** It should definitely be the time for us to see the instructor and get assistance to understand the materials and to practice MORE.

| Scores | 10 | 9 | 8 | 7 | 6 | $\leq 5$ |
|---|---|---|---|---|---|---|
| Frequency | 2 | 2 | 9 | 5 | 8 | 4 |
| Percentage | 6.7 | 6.7 | 30.0 | 16.7 | 26.7 | 13.2 |

**Discussions and Comments**

One of the purposes of this quiz is to see if we can differentiate sequences and series. Most of us (above 60%) knows the difference and use appropriate tools accordingly. If we have made a mistake in this quiz because we could not tell the difference between series and sequences, we will learn better and make no mistakes in the exams.
(1) **Problem 1:** As the limit of sequence requires the use of L'Hospital Rule, correctly computing the derivatives would be important. Some of the errors found here are errors computing the derivative, algebra in simplifying the fractions. However, it is a good thing to see that majority of us did make no errors in the computation of the limits.

(2) **Problem 2:** This is an easy exercise to let us to see the power of divergency test. Most of the errors occurred when some of us considered this is a sequence instead of a series.
(3) **Problem 3:** This is the trouble some exercise for many. Errors occurred when we are trying to remember what we did in class instead of trying to understand why we did it. The good side is that many have successfully model the problem into a geometric series. But few could compute the geometric series correctly to write the number as a ratio of integers.