Math 156 Fall 2015 Quiz 5

Name:

1: Find the length of the curve $y = \dfrac{x^2}{4} - \ln(\sqrt{x})$, $1 \leq x \leq 2$.

Solution: Use $\ln(\sqrt{x}) = \frac{1}{2}\ln(x)$ to compute $y' = \frac{x}{2} - \frac{1}{2x}$. Thus

$$\begin{aligned} 1 + \left(\frac{dy}{dx}\right)^2 &= 1 + \frac{x^2}{4} - 2 \cdot \frac{x}{2} \cdot \frac{1}{2x} + \frac{1}{4x^2} = 1 + \frac{x^2}{4} - \frac{1}{2} + \frac{1}{4x^2} \\ &= \frac{x^2}{4} + \frac{1}{2} + \frac{1}{4x^2} = \left(\frac{x}{2} + \frac{1}{2x}\right)^2 \end{aligned}$$

It follows that

$$\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{\left(\frac{x}{2} + \frac{1}{2x}\right)^2} = \frac{x}{2} + \frac{1}{2x}.$$

Hence the length is

$$\int_1^2 \left(\frac{x}{2} + \frac{1}{2x}\right) dx = \left[\frac{x^2}{4} + \frac{\ln(x)}{2}\right]_1^2 = 1 + \frac{\ln(2)}{2} - \frac{1}{4} = \frac{3}{4} + \frac{\ln(2)}{2}.$$

2: Find an integral to compute the length of the curve $y = \ln(\sec(x))$, $0 \leq x \leq \pi/4$. **Do not evaluate the integral.**

Solution: Compute $y' = \frac{\sec(x)\tan(x)}{\sec(x)} = \tan(x)$. $\sqrt{1 + (y')^2} = \sqrt{1 + \tan^2(x)} = \sec(x)$. Hence the integral computing the length is equal to

$$\int_0^{\frac{\pi}{4}} \sec(x) dx.$$

3: Find an integral to compute the length of the curve $x = y^2 - 2y$, $0 \leq y \leq 2$. **Do not evaluate the integral.**

Solution: Compute $x' = 2y - 2$. Hence $\sqrt{1 + (x')^2} = \sqrt{1 + 4y^2 - 8y + 4} = \sqrt{4y^2 - 8y + 5}$. Hence the integral computing the length is equal to

$$\int_0^2 \sqrt{4y^2 - 8y + 5} dy.$$