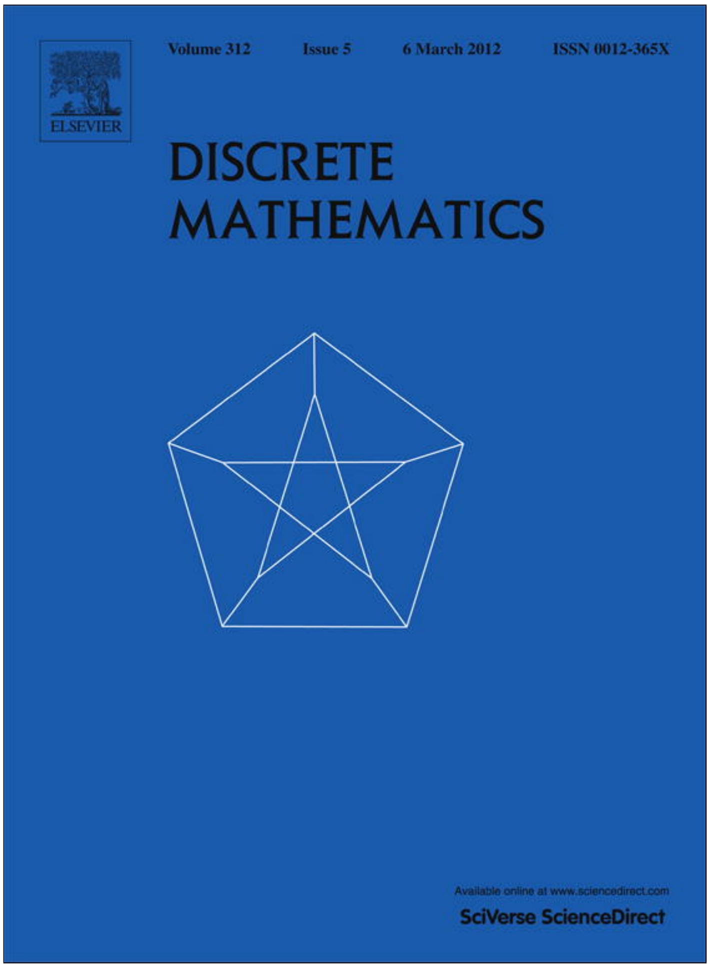
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Spanning subgraph with Eulerian components

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ABSTRACT

A graph is k-supereulerian if it has a spanning even subgraph with at most k components. We show that if G is a connected graph and G' is the (collapsible) reduction of G, then G is k-supereulerian if and only if G' is k-supereulerian. This extends Catlin's reduction theorem in [P.A. Catlin, A reduction method to find spanning Eulerian subgraphs, J. Graph Theory 12 (1988) 29–44]. For a graph G, let $F(G)$ be the minimum number of edges whose addition to G create a spanning supergraph containing two edge-disjoint spanning trees. We prove that if G is a connected graph with $F(G) \leq k$, where k is a positive integer, then either G is k-supereulerian or G can be contracted to a tree of order $k+1$. This is a best possible result which extends another theorem of Catlin, in [P.A. Catlin, A reduction method to find spanning Eulerian subgraphs, J. Graph Theory 12 (1988) 29–44]. Finally, we use these results to give a sufficient condition on the minimum degree for a graph G to be k-supereulerian.

1. Introduction

We use [1] for terminology and notation not defined here and consider only loopless finite graphs.

For $S \subseteq V(G)$, we denote by $G[S]$ the subgraph induced by S. In this paper, we also consider the subgraph induced by a set of edges. For $F \subseteq E(G)$, the subgraph H defined by $V(H) = V(F)$ and $E(H) = F$ is said to be the subgraph induced by F, and is denoted by $G[F]$. When we simply say an "induced subgraph", it means the subgraph induced by a set of vertices. A graph is *trivial* if it has only one vertex.

Let $O(G)$ denote the set of all odd-degree vertices of G. A *Eulerian graph* is a connected graph G with $O(G) = \emptyset$. A graph is *supereulerian* if it has a spanning Eulerian subgraph. A graph H is *collapsible* if for every even set $X \subseteq V(H)$, there is a spanning connected subgraph H_X of H such that $O(H_X) = X$. We regard K_1 as supereulerian and collapsible. We use $\mathcal{CL}$ and $\mathcal{SL}$ to denote the families of collapsible graphs and supereulerian graphs, respectively. Clearly, $\mathcal{CL} \subset \mathcal{SL}$ (see [6]).

For a graph G with a connected subgraph H, the *contraction* G/H is the graph obtained from G by replacing H by a new vertex v_H, such that the number of edges in G/H joining any $v \in V(G) - V(H)$ to v_H in G/H equals the number of edges joining v in G to H (v_H is called the *image* of H). Likewise, for a graph G and an edge set $E \subseteq E(G)$, G/E denotes the graph obtained from G by contracting the edges of E and deleting any resulting loops.

In [2], Catlin showed that any graph G has a unique collection of pairwise vertex-disjoint maximal collapsible subgraphs $H_1, H_2, \ldots, H_c$ such that $\bigcup_{i=1}^{c} V(H_i) = V(G)$. The *reduction* of G, denoted by G', is the graph obtained from G by contracting

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each H_i $(1 \leq i \leq c)$ to a single vertex. A graph G is *reduced* if $G = G'$ (see [2]). For surveys of work on supereulerian graphs, see [3,6].

A well-known theorem of Catlin is the following.

Theorem 1 *([2]). Let G be a connected graph and H be a collapsible subgraph of G. Then*

(a) *G is supereulerian if and only if G/H is supereulerian;*
(b) *G is supereulerian if and only if its reduction G' is supereulerian.*

This is a powerful result to study the existence of spanning and dominating Eulerian subgraphs. A graph G is *k-supereulerian* if it has a spanning even subgraph with at most k components. Obviously, 1-supereulerian graph is supereulerian. In Section 2, we extend Theorem 1 to k-supereulerian graphs, and prove the following result.

Theorem 2. *Let G be a connected graph and G' be the reduction of G. Then G is k-supereulerian if and only if G' is k-supereulerian.*

A spanning subgraph of a graph is a *factor*. An *even factor* of G is a spanning subgraph of G in which every vertex has even positive degree, and a 2-*factor* of G is an even factor in which every vertex has degree 2. In [11], the structure of even factors in claw-free graphs was studied.

Theorem 3 *([11]). Every simple claw-free graph G of order n with $\delta(G) \geq 3$ has an even factor with at most $\max\{1, \lfloor \frac{2n-2}{7} \rfloor\}$ components.*

Obviously, if G has an even factor with at most k components, then G is k-supereulerian whereas the converse is not true in general (for example, a tree with k vertices is k-supereulerian, but it has no even factor). By an observation of [11], we know that if G has an even factor with at most k components, then $L(G)$ has a 2-factor with at most k components. For more related results, see [7,8].

Now we extend the sufficiency of (a) in Theorem 1 to graphs with even factors.

Theorem 4. *Let H be a collapsible subgraph of G. If G/H has an even factor with exactly k components, then G has an even factor with exactly k components.*

Proof. Suppose G/H has an even factor F with exactly k components $F_1, F_2, \ldots, F_k$. Since v_H, the image of H, is a single vertex, without loss of generality, we can assume that $v_H \in V(F_1)$. Let $V_1 = V(G) \setminus (V(F_2) \cup V(F_3) \cup \cdots \cup V(F_k))$. Then F_1 is a spanning Eulerian subgraph of $G[V_1]/H$, and hence $G[V_1]/H$ is a nontrivial supereulerian subgraph, which implies that $G[V_1]$ is nontrivial. By (a) of Theorem 1, $G[V_1]$ is supereulerian. Since every nontrivial supereulerian graph has a spanning Eulerian subgraph as an even factor, $G[V_1]$ has an even factor F_1' with exactly one component. Then $F_1' \cup F_2 \cup \cdots \cup F_k$ is an even factor of G with exactly k components. This completes the proof of Theorem 4. □

For a graph G, define $F(G)$ to be the minimum number of edges that must be added to a graph, in order to obtain a spanning supergraph that has two edge-disjoint spanning trees. Tutte [10] and Nash-Williams [9] characterized the graphs having k edge-disjoint spanning trees, for any given k.

Another theorem of Catlin in [2] states that, if $F(G) \leq 1$, then either G is supereulerian, or $F(G) = 1$ and G has a cut edge. In this paper, we extend this result to graphs with $F(G) > 1$, which will be proved in Section 3.

Theorem 5. *Let k be a positive integer and G be a connected graph. If*

$$F(G) \leq k,$$

then exactly one of the following holds:

(a) *G is k-supereulerian;*
(b) *G can be contracted to a tree of order $k + 1$.*

This result is best possible. Let G be the graph obtained from $K_{2,t}$ (where t is odd and $t > 1$) by the addition of k pendant vertices, then (a) holds with equality: $F(G) = 2+k$, and any graph Γ satisfying (a) of Theorem 5 has exactly $2+k$ components (the k pendant vertices, $K_{2,t} - v$ and v, where v is a vertex of $K_{2,t}$ with degree 2).

An analogous result (a similar sufficient condition for G to have a spanning connected subgraph having few vertices of odd degree, unless G could be contracted to a tree of a certain size) was given by Catlin [4].

In [2], Catlin proved the following theorem.

Theorem 6 *([2]). Let G be a 2-edge-connected simple graph on n vertices. Let $b \in \{4, 5\}$. If*

$$\delta(G) \geq n/b - 1 \tag{1.1}$$

and if $n > 4b$, then exactly one of the following holds:

(a) *The equality holds in (1.1), and G is contractible to $K_{2,b-2}$ ($b \in \{4, 5\}$), such that the preimage of each vertex of $K_{2,b-2}$ is a collapsible subgraph of G on exactly n/b vertices;*

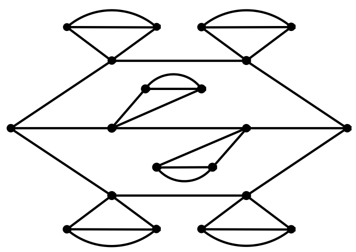

Fig. 1. A 3-supereulerian graph G which is not 2-supereulerian.

(b) $b = 5$ *and* G *is supereulerian;*
(c) $b = 4$ *and* G *is collapsible.*

As a corollary of Theorem 6, we have the following.

Corollary 7. *Let* G *be a* 2*-edge-connected simple graph on* n *vertices.*

(a) *If* $\delta(G) \geq n/4 - 1$ *and if* $n > 16$*, then* G *is* 1*-supereulerian.*
(b) *If* $\delta(G) \geq n/5 - 1$ *and if* $n > 20$*, then* G *is* 2*-supereulerian.*

In order to extend Corollary 7, by applying Theorem 5, we obtain the following result, which will be proved in Section 4.

Theorem 8. *Let* G *be a* 2*-edge-connected graph on* n *vertices. If* $\delta(G) \geq n/(3+k) - 1$ *and* $n > 4(3+k)$*, then* G *is* k*-supereulerian.*

For $k \in \{1, 2\}$, the bound $n > 4(3 + k)$ is sharp: for $k = 1$, see [2]; for $k = 2$, see Fig. 1 ($n = 4(3 + 2)$ and $\delta(G) = n/(3 + 2) - 1$, but G is not 2-supereulerian).

2. Contracting collapsible subgraphs does not change the *k*-supereulerian property

We shall prove Theorem 2 in this section. By the definition of G', it suffices to prove the following lemma, which means contracting a collapsible subgraph does not change the k-supereulerian property.

Lemma 9. *Let* G *be a connected graph and* H *be a collapsible subgraph of* G*. Then* G *is* k*-supereulerian if and only if* G/H *is* k*-supereulerian.*

Proof. Sufficiency. Suppose G/H is k-supereulerian, i.e., G/H has a spanning even subgraph F with $t\ (\leq k)$ components $F_1, F_2, \ldots, F_t$, then $(G/H)[V(F_i)]\ (1 \leq i \leq t)$ is supereulerian. Let v_H be the image of H. Without loss of generality, we can assume that $v_H \in V(F_1)$. Let $V_1 = V(G) \setminus (V(F_2) \cup V(F_3) \cup \cdots \cup V(F_t))$. Then $H \subseteq G[V_1]$, and hence, $G[V_1]/H = (G/H)[V(F_1)]$. By (a) of Theorem 1, $G[V_1] \in \mathcal{SL}$ and has a spanning even subgraph F_1'. Then $F_1' \cup F_2 \cup \cdots \cup F_t$ is a spanning even subgraph of G with $t\ (\leq k)$ components, i.e., G is k-supereulerian. □

Conversely, it suffices to prove the following claim.

Claim 1. *If* G *is* k*-supereulerian, then for any edge* $e = xy \in E(G)$*,* G/e *is also* k*-supereulerian.*

Proof of Claim 1. Suppose G is k-supereulerian, i.e., G has a spanning even subgraph F with at most k components. Let F' be the graph obtained from F by contracting e to a single vertex v_e. Then

$$d_{F'}(v_e) = \begin{cases} d_F(x) + d_F(y) - 2 - 2\ell, & \text{if } e \in E(F) \\ d_F(x) + d_F(y) - 2\ell, & \text{if } e \notin E(F) \end{cases}$$

where ℓ is the number of resulting loops. Since F is an even graph, $d_F(x)$ and $d_F(y)$ are both even. Hence, $d_{F'}(v_e)$ is even. Note that $d_{F'}(v) = d_F(v)$ is even for each $v \neq v_e$. F' is also an even graph. Since contracting an edge cannot increase the number of components of F, F' is a spanning even subgraph of G/e with at most k components, i.e., G/e is k-supereulerian. Our claim is proved.

This completes the proof of Lemma 9. □

Corollary 10. *Let* G *be a connected graph. Then*

$$\min\{k \mid G \text{ is } k\text{-supereulerian}\} = \min\{k \mid G/H \text{ is } k\text{-supereulerian}\},$$

for any collapsible subgraph H *of* G*.*

Proof. Let $k_1 = \min\{k|\, G \text{ is } k\text{-supereulerian}\}$ and $k_2 = \min\{k|\, G/H \text{ is } k\text{-supereulerian}\}$. We prove $k_1 \le k_2$ by way of contradiction first. Suppose $k_1 > k_2$, which implies that G has a collapsible subgraph H such that G/H is k_2-supereulerian. Then by Lemma 9, G is also k_2-supereulerian, which contradicts the minimality of k_1. Similarly, we have $k_1 \ge k_2$. So $k_1 = k_2$. □

Corollary 10 has an immediate consequence.

Corollary 11. *Let G be a connected graph and G' be the reduction of G. Then*

$$\min\{k|\, G \text{ is } k\text{-supereulerian}\} = \min\{k|\, G' \text{ is } k\text{-supereulerian}\}.$$

3. A sufficient condition for k-supereulerian graphs involving $F(G)$

The main result of this section is Theorem 5. We start with a theorem of Catlin [2] and two lemmas, which will be used in the proof of Theorem 5.

Theorem 12 ([2]). *Let G be a graph. If $F(G) \le 1$, then G is connected and exactly one of the following holds:*

(a) *G is supereulerian.*
(b) *$F(G) = 1$ and G has a cut-edge.*

Lemma 13. *Let G be a connected graph with $F(G) = k$. Then G has edge-disjoint spanning forests T and U such that T is a tree and U has exactly $k + 1$ components. Moreover, there is a subset $E \subseteq E(T)$ with $|E| = k$ such that*

(a) *$U + E$ is a spanning tree of G; and*
(b) *E contains all cut-edges of G.*

Proof. First, we prove the existence of T and U. Let $\mathscr{X} = \{X_1, X_2, \ldots, X_m\}$ be the family of minimal edge sets X_i such that $G + X_i$ has two edge-disjoint spanning trees T_i and T_i^* and such that $|E(T_i) \cap X_i| \le |E(T_i^*) \cap X_i|$. Without loss of generality, we may assume that $|E(T_1) \cap X_1| = \min\{|E(T_i) \cap X_i| \,:\, X_i \in \mathscr{X}\}$.

If $|E(T_1) \cap X_1| \ge 1$, then one can find a pair of edges $e \in E(T_1) \cap X_1$ and $e' \in E(G - E(T_1))$ such that $T_0 = T_1 - e + e'$ is a spanning tree of $G + X_1$. Then $e' \in E(T_1^*)$: for otherwise, $G + (X_1 \setminus \{e\})$ has two edge-disjoint spanning trees T_0 and T_1^* such that $|X_1 \setminus \{e\}| < |X_1|$, a contradiction. Hence, there is an edge $e'' \notin E(G + X_1)$ such that $T_0^* = T_1^* - e' + e''$ is a spanning tree of $G + X_0$, where $X_0 = (X_1 \cup \{e''\}) \setminus \{e\}$. Note that $T_0 \subset G + X_0$. Then $G + X_0$ has two edge-disjoint spanning trees T_0 and T_0^* with $|X_0 \cap E(T_0)| < |X_1 \cap E(T_1)|$, contrary to the choice of X_1.

Hence, $|E(T_1) \cap X_1| = 0$, i.e., $T = T_1$ is a spanning tree of G. Then $U = T_1^* - X_1$ is a spanning forest of G with exactly $k+1$ components. The existence of T and U is proved.

Moreover, since T is a spanning tree of G, there is a subset $E \subseteq E(T)$ with $|E| = k$ such that $U + E$ is a spanning tree of G. Note that T and $U + E$ contain all cut-edges of G, and T and U are edge-disjoint. E contains all cut-edges of G. □

Lemma 14. *Let G be a connected graph with $F(G) = k$. Define T, U and E as in Lemma 13. If E contains an edge e that is not a cut-edge of G, then each of the following holds for $G/(E - e)$.*

(a) *$F(G/(E - e)) \le 1$.*
(b) *$G/(E - e)$ is 2-edge-connected.*
(c) *$G/(E - e)$ is supereulerian.*

Proof. Since E has an edge e that is not a cut-edge of G, by (b) of Lemma 13, $E - e$ contains all cut-edges of G. Hence, $G/(E - e)$ is 2-edge-connected, and then (b) of Lemma 14 holds. Note that $G/(E - e)$ has spanning trees $T/(E - e)$ and $U + e$, and the only edge in both of these trees is e, hence (a) holds. By Theorem 12, and by (a), (b) of Lemma 14, $G/(E - e)$ is supereulerian. Hence, (c) holds. □

Now we prove Theorem 5.

Proof of Theorem 5. If $F(G) = 0$, then by Theorem 12, G has a spanning Eulerian subgraph, and so (a) of Theorem 5 holds with $F(G) = 0$. Thus, suppose

$$F(G) = k \ge 1. \tag{3.1}$$

By (3.1) and Lemma 13, G has edge-disjoint spanning forests T and U such that T is a tree, U has exactly $k+1$ components, and $E(T)$ has a subset E with $|E| = k \ge 1$ satisfying (a) and (b) of Lemma 13.

If each edge in E is a cut-edge of G, then (b) of Theorem 5 holds.

Henceforth, suppose that E has an edge e that is not a cut-edge of G. Then the contraction $G/(E - e)$ has the properties (a)–(c) of Lemma 14. By (c) of Lemma 14, $G/(E - e)$ has a spanning Eulerian subgraph Γ_0 (say). Let Γ be the subgraph of G induced by $E(\Gamma_0)$ and $H_1, H_2, \ldots, H_t$ $(t < k)$ be the components of $G[E - e]$. When all H_i's $(1 \le i \le t)$ in G are contracted to distinct vertices, Γ (in G) becomes the Eulerian subgraph Γ_0 of $G/(E - e)$, and hence, Γ must have at most $t + 1$ components.

Since each H_i $(1 \le i \le t)$ is contracted to a vertex (in $G/(E-e)$) whose degree in Γ_0 is even, each H_i $(1 \le i \le t)$ contains a set R_i (say) of an even number (possibly zero) of vertices of odd degree in Γ. We claim that H_i has a subgraph Γ_i (say) with R_i as its odd-degree vertices. Note that H_i has paths $P_1, P_2, \ldots, P_s$ that join the $2s$ vertices of R_i in pairs, where $2s = |R_i|$. Γ_i is induced by those edges of H_i lying in an odd number of the paths P_i $(1 \le i \le s)$. This proves the claim about Γ_i. Define

$$G_0 = G\left[E(\Gamma)\bigcup\left(\bigcup_{i=1}^{t}E(\Gamma_i)\right)\right].$$

Then all vertices of G_0 have even degree. When all H_i's $(1 \le i \le t)$ are contracted to distinct vertices, the various components of Γ are attached to form the connected graph Γ_0 in $G/(E-e)$. Since Γ is a spanning subgraph of G_0, the graph G_0 has no more components than Γ does. Thus, G_0 has at most $t+1$ components, which are all Eulerian. By $t \le k-1$, (a) of Theorem 5 holds.

Since (a) and (b) are mutually exclusive, Theorem 5 holds. □

Corollary 15. *Let G be a 2-edge-connected graph. If $F(G) \le k$, then G is k-supereulerian.*

4. An application of Theorem 5

In this section, we shall prove Theorem 8, which is an application of Theorem 5. First, we prepare two results.

Let $a(G)$ denote the *edge arboricity* of G, i.e., the minimum number of edge-disjoint forests whose union equals G. The following theorem of Catlin gives an equation involving $F(G)$, m (the number of edges) and n (the order of G).

Theorem 16 ([5]). *If G is reduced, then $a(G) \le 2$; if $a(G) \le 2$ and if G has n vertices and m edges, then*

$$F(G) + m = 2n - 2.$$

Let $D_i(G) = \{v \in V(G) | \, d_G(v) = i\}$.

Lemma 17. *Let G be a 2-edge-connected reduced graph on n vertices and m edges, and k be a positive integer. If*

$$|D_2(G)| + |D_3(G)| \le 3 + k,$$

then either G is supereulerian or $F(G) \le k$.

Proof. Suppose that $F(G) > k$ and let $d_i = |D_i(G)|$. By Theorem 16, $F(G) = 2n - m - 2$. Since G is 2-edge-connected, we have $n = \sum_{i\ge 2} d_i$ and $2m = \sum_{i \ge 2} id_i$. Then

$$4n - 2m - 4 = 4\sum_{i\ge 2} d_i - \sum_{i\ge 2} id_i - 4 = 2d_2 + d_3 - \sum_{i\ge 5}(i-4)d_i - 4,$$

by $F(G) > k$,

$$2d_2 + d_3 - \sum_{i\ge 5}(i-4)d_i - 4 > 2k.$$

As $d_2 + d_3 \le 3 + k$,

$$d_2 + 3 + k - \sum_{i\ge 5}(i-4)d_i > 2k + 4,$$

and so

$$d_2 - \sum_{i\ge 5}(i-4)d_i > k + 1,$$

or

$$3 + k - (d_3 + d_5 + 2d_6 + \cdots (i-4)d_i + \cdots) \ge d_2 + d_3 - (d_3 + d_5 + 2d_6 + \cdots (i-4)d_i + \cdots) > k + 1.$$

It follows that

$$d_3 + d_5 + 2d_6 + \cdots (i-4)d_i + \cdots < 2.$$

This implies that $\Delta(G) \le 5$, and hence, $d_3 + d_5 < 2$.

Since the number of odd degree vertices in any graph must be even, we have $(d_3, d_5) = (0, 0)$. Hence, G is Eulerian. The proof of Lemma 17 is finished. □

We now prove Theorem 8.

Proof of Theorem 8. Let G' denote the reduction of G and $D_i = D_i(G')$.

Since $\delta(G) \geq n/(3+k) - 1$ and $n > 4(3+k)$, we have $\delta(G) \geq 4$. Then by the degree condition, we prove the following claim first. □

Claim 2. $|D_2| + |D_3| \leq 3 + k$.

Proof of Claim 2. Let c denote the order of G'. Index the vertices $v_i \in V(G')$ such that

$$d(v_1) \leq d(v_2) \leq \cdots \leq d(v_c), \tag{4.1}$$

and define the induced subgraph H_i to be the preimage of v_i $(1 \leq i \leq c)$ in the contraction: $G \to G'$. For each $v_i \in D_2 \cup D_3$, $d(v_i) < \delta(G)$, implying that some $x_i \in V(H_i)$ has $N(x_i) \subseteq V(H_i)$, and so $|V(H_i)| \geq d(x_i) + 1 \geq \delta(G) + 1 \geq n/(3+k)$. Denote $|D_2| + |D_3| = b$. If $b > 3 + k$, then

$$n = \sum_{i=1}^{c} |V(H_i)| \geq \sum_{i=1}^{b} |V(H_i)| \geq b \cdot \frac{n}{3+k} > (3+k)\frac{n}{3+k} = n,$$

a contradiction. Our claim is proved.

Then by Lemma 17 and Claim 2, either G' is supereulerian, and hence, G' is k-supereulerian; or $F(G') \leq k$, by Corollary 15, G' is k-supereulerian. By Theorem 2, G is k-supereulerian. □

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