

Supereulerian Graphs: A Survey

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ABSTRACT

A graph is *supereulerian* if it has a spanning eulerian subgraph. There is a reduction method to determine whether a graph is supereulerian, and it can also be applied to study other concepts, e.g., hamiltonian line graphs, a certain type of double cycle cover, and the total interval number of a graph. We outline the research on supereulerian graphs, the reduction method, and its applications.

1. NOTATION

We follow the notation of Bondy and Murty [22], with these exceptions: a *graph* has no loops, but multiple edges are allowed; the trivial graph K_1 is regarded as having infinite edge-connectivity; and the symbol E will normally refer to a subset of the edge set $E(G)$ of a graph G , not to $E(G)$ itself.

The graph of order 2 with 2 edges is called a *2-cycle* and denoted C_2 . Let H be a subgraph of G . The *contraction* G/H is the graph obtained from G by contracting all edges of H and deleting any resulting loops. For a graph G , denote

$$O(G) = \{\text{odd-degree vertices of } G\}.$$

A graph with $O(G) = \emptyset$ is called an *even graph*. A graph is *eulerian* if it is connected and even. We call a graph G *supereulerian* if G has a spanning eulerian subgraph. Regard K_1 as supereulerian. Denote

$$\mathcal{SL} = \{\text{supereulerian graphs}\}.$$

Let G be a graph. The *line graph* of G (called an edge graph in [22]) is denoted $L(G)$, it has vertex set $E(G)$, where $e, e' \in E(G)$ are adjacent vertices in $L(G)$ whenever e and e' are adjacent edges in G .

Let \mathcal{S} be a family of graphs, let G be a graph, and let $k \geq 0$ be an integer. If there is a graph $G_0 \in \mathcal{S}$ such that G can be obtained from G_0 by removing at most k edges, then G is said to be *at most k edges short* of being in \mathcal{S} . For a graph G , we write $F(G) = k$ if k is the least nonnegative integer such that G is at most k edges short of having 2 edge-disjoint spanning trees.

2. INTRODUCTION

In this paper, we survey the research on the graph family \mathcal{SL} of supereulerian graphs and on an associated reduction method for determining membership in \mathcal{SL} . Certain related problems are also discussed.

Three old and important results on this topic are theorems of Euler [63], Harary and Nash-Williams [81], and Jaeger [85].

Theorem 2.1 ([63], [83], [142], [128], [134], [110]). If G is a connected nontrivial graph, then these are equivalent:

- (a) G has a closed trail that uses each edge exactly once.
- (b) G is eulerian.
- (c) G is an edge-disjoint union of cycles.
- (d) The number of sets (including the empty set) of edges of G , each of which is contained in a spanning tree of G , is odd.
- (e) Every edge of G lies on an odd number of cycles. ■

The first complete published proof of (a) \Leftrightarrow (b) of Theorem 2.1 was due to C. Hierholzer [83] in 1873. Conditions (c), (d), and (e), respectively, were due to Veblen [142], Shank [128], and McKee's [110] modification of Toida's Theorem [134]. For a history, see [18], [126], and [147], and for a review of related literature see [67], [101], and [102].

By definition and by (a) \Leftrightarrow (b), a nontrivial graph G is supereulerian whenever G has a spanning closed trail. A trail T in G is called *dominating* if each edge of G is either in T or adjacent to an edge in T . If a connected graph has no dominating trail, then it is not supereulerian. The *subdivision graph* $S_1(G)$ is obtained from G by inserting a new vertex into each edge.

Theorem 2.2 (Harary and Nash-Williams [81]). Let G be a graph not a star. Then

- (a) $L(G)$ is hamiltonian if and only if G has a dominating closed trail;
- (b) $L(S_1(G))$ is hamiltonian if and only if $G \in \mathcal{SL}$. ■

Lesniak-Foster and Williamson [104] and S.-M. Zhan [149] noted that a graph G has a dominating open trail if and only if $L(G)$ has a hamilton path.

Theorem 2.3 (Jaeger [85]). Let G be a graph and let $E \subseteq E(G)$. There is an even subgraph H of G with $E \subseteq E(H)$ if and only if E contains no bond of G with odd cardinality. ■

When E is the set of edges of one of two edge-disjoint spanning trees of G , then E contains no bond of G , and so Theorem 2.3 implies:

Corollary 2.3A (Jaeger [85]). If G has two edge-disjoint spanning trees, then G is supereulerian. ■

Corollary 2.3B (Boesch, Suffel, and Tindell [20]). Let G be a simple graph. There is a simple eulerian spanning supergraph of G if and only if G is not spanned by an odd complete bipartite graph. ■

Corollary 2.3B was extended in [129].

A 3-regular graph is supereulerian if and only if it is hamiltonian. Pulleyblank [125] noted that since the hamiltonian problem is NP-complete for such graphs [74], so is the supereulerian problem. The hamiltonian line graph problem is also NP-complete [17].

Recall that the *vertex arboricity* of a graph G is the minimum number of classes into which $V(G)$ can be partitioned, such that each class induces a forest in G . The following was announced by Jaeger [85], and a proof is in [80].

Theorem 2.4 ([85], [80]). A planar graph G is supereulerian if and only if its planar dual G^* has vertex arboricity at most 2.

Chartrand and Kronk [50] had previously proved that a simple planar graph G has vertex arboricity at most 3, and they cited the planar dual of Tutte's counterexample [136] to Tait's conjecture [131] (that a bridgeless planar 3-regular graph is supereulerian), to show that this bound is sharp when G is 3-regular. Bosak [21] gave a counterexample of order 38 to Tait's conjecture.

3. REDUCTION THEOREMS

Let G be a graph and let H be a connected subgraph of G . Catlin [32] obtained a sufficient condition for this equivalence to hold:

$$G \in \mathcal{SL} \iff G/H \in \mathcal{SL}. \tag{1}$$

By repeated applications of (1), the problem of determining whether $G \in \mathcal{SL}$ can often be reduced to determining whether a much smaller graph is in \mathcal{SL} .

Call a graph H *collapsible* if for every even set $X \subseteq V(H)$ there is a spanning connected subgraph H_X of H such that $O(H_X) = X$. Denote

$$\mathcal{CL} = \{\text{collapsible graphs}\}.$$

For example, K_1 , C_2 , and C_3 are collapsible, but $C_4 \notin \mathcal{CL}$. Also, $\mathcal{CL} \subset \mathcal{PL}$, and any collapsible graph is 2-edge-connected.

Theorem 3.1 (Catlin [32]). Let H be a subgraph of G . If $H \in \mathcal{CL}$, then

$$G \in \mathcal{PL} \iff G/H \in \mathcal{PL}$$

and

$$G \in \mathcal{CL} \iff G/H \in \mathcal{CL}. \quad \blacksquare$$

A more general result is Theorem 3 of [32]. This result and a criterion of Cai [27] are harder to state.

Conjecture (Catlin [39]). Let H be a graph. If $H \notin \mathcal{CL}$, then there is a supergraph G of H such that

$$G \in \mathcal{PL} \not\iff G/H \in \mathcal{PL}. \quad \blacksquare \tag{2}$$

4. THE REDUCTION METHOD

A general reduction method is based upon Theorem 3.1. To apply Theorem 3.1, it is useful to know a large class of collapsible graphs. The next result gives a large class, and since $\mathcal{CL} \subset \mathcal{PL}$, it improves Corollary 2.3A:

Theorem 4.1 [32]. If G is at most one edge short of having two edge-disjoint spanning trees, then exactly one of these holds:

- (a) $G \in \mathcal{CL}$;
- (b) G has a single cut-edge. \blacksquare

To improve Theorem 4.1, Catlin [40] conjectured that any 2-edge-connected graph that is at most 2 edges short of having two edge-disjoint spanning trees is either collapsible or contractible to $K_{2,t}$ for some $t \geq 2$.

Let $t:V(G) \rightarrow \mathbf{Z}$. A subset $F \subseteq E(G)$ is called a t -join if at each vertex $v \in V(G)$, the number of incident edges in F is congruent to $t(v) \pmod{2}$. It is easily checked that any spanning tree of G contains a t -join, for any t with $t(v)$ odd at evenly many vertices v . It follows that if G has two edge-disjoint spanning trees, then $G \in \mathcal{CL}$. This is slightly weaker than Theorem 4.1.

Theorem 4.2 [32]. Let H_1 and H_2 be collapsible subgraphs of a graph G . If $V(H_1) \cap V(H_2) \neq \emptyset$, then $H_1 \cup H_2 \in \mathcal{CL}$. ■

Any graph G has a unique set H_1, H_2, \dots, H_c of pairwise-disjoint maximal collapsible subgraphs, by Theorem 4.2. Since $K_1 \in \mathcal{CL}$, each vertex of G is in some H_i ($1 \leq i \leq c$). Let G' denote the graph obtained from G by contracting each H_i ($1 \leq i \leq c$) to a distinct vertex. We call G' the *reduction* of G , and a graph is called *reduced* if it is the reduction of some graph. Thus, $G \in \mathcal{CL}$ if and only if $G' = K_1$.

Theorem 4.3 [32]. A graph is reduced if and only if it has no nontrivial collapsible subgraph. ■

Examples of reduced graphs include K_1, K_2 , stars, $K_{2,t}$ ($t \in \mathbb{N}$), and the Petersen graph (for more on this graph, see [52]). Also, a simple graph G is reduced if the contraction of some edge of G yields $K_{2,t}$ ($t \in \mathbb{N}$). H.-J. Lai [99,100] showed that these are the only reduced graphs of diameter at most 2. Using this and Theorem 2.2, one can obtain Veldman's result [145] that if G has diameter at most 2 and order at least 4, then $L(G)$ is hamiltonian.

Let G be a graph. If H is a connected reduced induced subgraph of G , then G is reduced if and only if G/H is reduced [39]. For another construction of reduced graphs, see [41].

By repeated applications of Theorem 3.1,

$$G \in \mathcal{PL} \iff G' \in \mathcal{PL}. \tag{3}$$

Let G be a graph. The *edge arboricity* $a_1(G)$ of G is the minimum number of forests in G whose union contains G . Nash-Williams [112] proved

$$a_1(G) = \max_{H \subseteq G} \left\lceil \frac{|E(H)|}{|V(H)| - 1} \right\rceil,$$

where the maximum is taken over all nontrivial subgraphs H of G .

Theorem 4.4 [32]. If G is a reduced graph, then $a_1(G) \leq 2$, any cycle of G has length at least 4, and $\delta(G) \leq 3$. ■

If $G \in \mathcal{CL}$, then G has three spanning trees T_1, T_2, T_3 such that each edge of G is in at most two members of $\{T_1, T_2, T_3\}$ [39]. Since $K_{2,t}$ ($t \in \mathbb{N}$) is reduced, the converse is false. However, the result is best possible, since there are infinitely many collapsible graphs G of girth 4 with $|E(G)| = 3(|V(G)| - 1)/2$ (see [33,35]).

Different reduction methods to treat 4-cycles appear in [35] and [98].

Extending a result of Chen [55], Chen and Lai [58] proved that the reduction of a connected simple graph G with $\delta(G) \geq 3$ and order at most 11 is

either K_1 , K_2 , or the Petersen graph. If G is a connected simple graph of order at most 13 with $\delta(G) \geq 3$, then either $G \in \mathcal{SL}$ or the reduction of G is K_2 , $K_{1,2}$, or the Petersen graph. Chen [53] gave a reduced graph G of order 14 to show that “13” is best possible: let G be a 3-regular graph containing a subgraph H , such that H is $K_{2,3}$ and G/H is the Petersen graph. Catlin [43] conjectured that any 3-edge-connected simple graph of order at most 17 is either supereulerian, or it is contractible to the Petersen graph.

5. SUFFICIENT CONDITIONS

Numerous sufficient conditions for $G \in \mathcal{SL}$ have been expressed in terms of lower bounds on degrees in G (see, e.g., [12], [26]–[28], [31], [32], [34], [36], [53], [56]–[59], [104]–[106], [120], [144], [148]). Some of these sufficient conditions even imply that G satisfies the hypothesis of Theorem 4.1, and the hypothesis of that theorem can be satisfied by graphs with far fewer edges.

Degree conditions are often excessively restrictive, and good sufficient conditions for a graph to be supereulerian can be found in terms of other parameters. Sufficient conditions for $G \in \mathcal{SL}$ have been expressed in terms of forbidden subgraphs, a requirement that each edge lies in a short cycle, or a lower bound on the number of edges (see, e.g., [6], [26], [29], [32], [44], [46], [98], [100], [118], [120]). Lai [98,100] showed that if a 2-connected graph G has $\delta(G) \geq 3$ and each edge lies in a cycle of length at most 4, then $G \in \mathcal{CL}$. Paulraja [118,120] had conjectured that such graphs are supereulerian. For more conditions, see [30] and [55], as well as Theorem 4.1.

Chen and Lai [57] proved that any reduced graph G of order n with $\delta(G) \geq 3$ has a matching of size at least $(n + 4)/3$. They used this (in [57] and [58]) to prove the following result, as well as some stronger results: If G is a 3-edge-connected graph and if $d(u) + d(v) \geq (n/5) - 2$ for every edge $uv \in E(G)$, then either $G \in \mathcal{SL}$ or equality holds and G is contractible to the Petersen graph. (This is related to a conjecture in [12].)

Sufficient conditions requiring relatively few edges are often expressed in terms of edge-connectivity or in terms of the number of edge-disjoint spanning trees (e.g., Corollary 2.3A and Theorems 5.2, 6.3, 6.4, and 6.9 below). The following relation between these properties follows from the theorem of Tutte [139] and Nash-Williams [111] characterizing graphs with k edge-disjoint spanning trees.

Theorem 5.1 (Catlin [39]). Let G be a graph and let $k \in \mathbb{N}$. Then G is $2k$ -edge-connected if and only if for any set E_k of k edges of G , the subgraph $G - E_k$ has at least k edge-disjoint spanning trees. ■

Corollary 5.1A ([123], [79], [95]). For any $k \in \mathbb{N}$, a $2k$ -edge-connected graph has k edge-disjoint spanning trees. ■

For $k = 2$, the “ \Rightarrow ” part of Theorem 5.1 was proved by Zhan [149], and the corollary was obtained by Jaeger ([86], Prop. 8(b)) when $k = 2$.

By Corollary 5.1A and Corollary 2.3A, any 4-edge-connected graph is supereulerian. These sufficient conditions were used in [19] and [73] with different probabilistic models to show that almost all graphs are supereulerian. Palmer [117] gave a direct proof.

Barnette conjectured [8] that any 3-regular 3-connected bipartite planar graph is supereulerian. Some 3-connected examples (Appendix III of [22], and [64]) show that the assumption of planarity is required. Goodey [77] obtained significant results for this problem, and an excellent account is found in Fleischner’s book [69]. Other partial results on Barnette’s conjecture have been obtained by Fleischner [65,68], by Fouquet and Thuillier [72], by Peterson [121], by Holton, Manvel, and McKay [84], and by Plummer and Pulleyblank [122]. Related conjectures are discussed in [70] and [72].

Several authors have studied closely related questions for which the reduction method still applies, such as whether a graph has a spanning trail (possibly open) (see [31], [47], [59], [144], [148], [149]), or such as this:

Theorem 5.2 [37]. Let k be a nonnegative integer and let G be a connected graph. If $F(G) \leq 2k + 1$, then exactly one of the following holds:

- (a) G has a spanning connected subgraph with at most $2k$ vertices of odd degree;
- (b) G can be contracted to a tree of order $2k + 2$ whose vertices all have odd degree. ■

6. APPLICATION: DOMINATING TRAILS

Any spanning trail is a dominating trail. Part (a) of Theorem 2.2 was an important motivation for studying dominating trails, because it gives a criterion for a line graph to be hamiltonian. By Corollary 5.1A, Corollary 2.3A and (a) of Theorem 2.2, the line graph of any 4-edge-connected graph is hamiltonian. Zhan [149] proved that the line graph of a 4-edge-connected graph is also hamiltonian connected. Thomassen [133] conjectured that every 4-connected line graph is hamiltonian. Equivalent conjectures are discussed in [70], and Zhan [150] proved Thomassen’s Conjecture for the case of 7-connected line graphs.

For other sufficient conditions for a graph to have a dominating trail, see [5]–[7], [9]–[15], [23]–[26], [45], [46], [49], [51], [53], [54], [59], [61], [62], [82], [90], [92], [96]–[98], [100], [103], [104], [108], [113]–[115], [119], [143]–[146], [148], [149]. The reduction method is not too useful for finding dominating cycles or paths, but it can be applied to find dominating trails.

Let G be a graph, let G' be its reduction, and let $\theta: V(G) \rightarrow V(G')$ be the mapping induced by the contraction $G \rightarrow G'$ defining G' . For any $v \in$

$V(G)$, v is called *trivial* if $|\theta^{-1}(v)| = 1$; v is called *nontrivial* otherwise. The following theorem was used in [45], [46], [97], [98], and [100].

Theorem 6.1 ([32], Theorem 8(vii)). Let G be a graph and let G' be the reduction of G . Then G has a dominating closed trail if and only if G' has a closed trail containing at least one vertex of each edge of G' and containing each nontrivial vertex of G' .

Dominating trails were used by Jünger, Reinelt, and Pulleyblank [89]. For any $s \in \mathbb{N}$, an s -partition of a graph G is a partition

$$E(G) = E_1 \cup E_2 \cup \cdots \cup E_k,$$

where $|E_i| = s$ ($1 \leq i \leq k - 1$), where $1 \leq |E_k| \leq s$, and where $G[E_i]$ is a connected subgraph ($1 \leq i \leq k$).

Theorem 6.2 ([89], Lemma 3.1). Any graph with a dominating trail has an s -partition, for all $s \in \mathbb{N}$. ■

Jünger, Reinelt, and Pulleyblank [89] used Theorem 6.2 and Corollary 2.3A (with a hypothesis of 4-edge-connectedness) to show

Theorem 6.3 [89]. Any 4-edge-connected graph has an s -partition, for all $s \in \mathbb{N}$. ■

They conjectured that any 3-edge-connected graph has an s -partition, for all $s \in \mathbb{N}$. Some 3-edge-connected graphs have no dominating trail, but Fleischner, Jackson, and Ash (see [68], Conjecture 25) conjectured that every 3-regular cyclically 4-edge-connected graph has a dominating cycle.

By Theorem 5.1 with $k = 2$, the next result improves Theorem 6.3:

Theorem 6.4. If a connected graph G is at most 3 edges short of having two edge-disjoint spanning trees, then exactly one of these holds:

- (a) G has an s -partition, for all $s \in \mathbb{N}$;
- (b) G has three cut edges that are not on one path, and none of them is incident with a vertex of degree 1. ■

Theorem 6.4 is proved by combining Theorem 6.2 with the case $k = 1$ of Theorem 5.2. The graph G of Figure 1 satisfies (b) of Theorem 6.4, and G violates (a). In Figure 1, each oval represents a nontrivial subgraph of G that has at least two edge-disjoint spanning trees and has the stated number of edges.

Next we define the total interval number $I(G)$ of a graph G . For $1 \leq i \leq n$, let v_i be a finite union of disjoint closed intervals of real num-

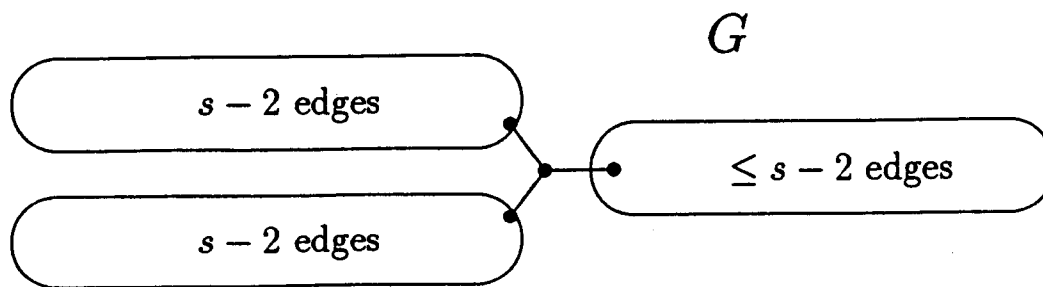


FIGURE 1. A graph with no s -partition.

bers. Denote by $|v_i|$ the number of intervals in v_i . Define a graph G with vertex set

$$V(G) = \{v_1, v_2, \dots, v_n\}$$

by regarding v_i and v_j ($i \neq j$) as adjacent in G if $v_i \cap v_j \neq \emptyset$. We say that $\{v_1, v_2, \dots, v_n\}$ is a *representation* of G . Griggs and West [78] and Andreae and Aigner [1] defined the *total interval number* $I(G)$ to be the minimum value of $\sum_{i=1}^n |v_i|$, among all representations of G . (The *interval number* of G , defined by Trotter and Harary [135], is different: it is the minimum of $\max_{1 \leq i \leq n} |v_i|$ over all representations of G .)

Andreae and Aigner [1] obtained an upper bound on $I(G)$ in terms of dominating trails, and Kratzke and West [93,94] noted that equality holds if G is triangle-free:

Lemma 6.5 ([1], [93], [94]). Let k be a nonnegative integer. If a connected graph G is at most k edges short of having a dominating trail, then

$$I(G) \leq |E(G)| + k + 1, \tag{4}$$

and if G is triangle-free, then equality holds in (4). ■

To apply Lemma 6.5, use this routine extension of Theorem 6.1:

Theorem 6.6. Let G be a graph and let G' be the reduction of G . Then G is at most k edges short of having a dominating trail if and only if G' is at most k edges short of having a trail containing at least one vertex of each edge of G' and containing each nontrivial vertex of G' . ■

Extending results of Andreae and Aigner [1] (who did the triangle-free case), Kostochka [91] and Kratzke and West [93,94] obtained various results, including

Theorem 6.7 ([93,94]). If G is a planar graph of order $n \geq 3$, then $I(G) \leq 2n - 3$. ■

Theorem 6.8 ([91,94]). If G is a connected graph with $m \geq 2$ edges, then $I(G) \leq (5m + 4)/4$. ■

It is routine to combine Lemma 6.5 with Theorem 5.2 to get this:

Theorem 6.9. Let G be a connected graph and let k be a nonnegative integer. If $F(G) \leq 2k + 1$, then

$$I(G) \leq |E(G)| + k + 1. \quad (5)$$

If also $k \geq 1$ and if equality holds in (5), then the reduction of G is $K_{1,2k+1}$, and each vertex of degree 1 in this $K_{1,2k+1}$ is a nontrivial vertex. ■

7. APPLICATION: CYCLE DOUBLE COVERS

A *cycle double cover* of a graph G is a collection \mathcal{C} of cycles in G (multiplicities allowed) such that each edge of G is in exactly two cycles in \mathcal{C} . Szekeres [130] and Seymour [127] conjectured that any graph with no cut edge has a cycle double cover. If G is 2-connected and planar, this conjecture is obvious: let \mathcal{C} be the facial cycles of some planar embedding. Alspach and Zhang [2], Goddyn [75,76], and Tarsi [132] have obtained significant partial results. See also [68] and [87]. Here, we merely note an application of the reduction method to a certain kind of cycle double cover related to \mathcal{SL} and \mathcal{CL} .

By (b) \Leftrightarrow (c) of Theorem 2.1, it is an equivalent conjecture that a graph G with no cut-edge has a collection \mathcal{E} of even subgraphs (multiplicities allowed), such that each edge of G is in exactly two members of \mathcal{E} . We call \mathcal{E} a *double cover of G by even graphs*. Celmins [48] and Preissmann [124] conjectured that any graph G with no cut edge has a double cover by at most five even graphs. This is best possible, because the Petersen graph requires five even graphs (all cycles) in a double cover.

Let \mathcal{S}_3 denote the family of graphs G with no cut edge, such that G has a double cover by at most three even graphs. Equivalently, $G \in \mathcal{S}_3$ whenever there is a partition $E(G) = E_1 \cup E_2 \cup E_3$ such that $O(G) = O(G[E_i])$ for $1 \leq i \leq 3$. Then $G[E_1 \cup E_2]$, $G[E_1 \cup E_3]$, and $G[E_2 \cup E_3]$ are the graphs that form a cycle double cover. In this section, we review results concerning the family \mathcal{S}_3 . We are specifically interested in \mathcal{S}_3 because it is related to \mathcal{SL} and because the reduction method for \mathcal{SL} applies similarly to \mathcal{S}_3 . Catlin and Lai (see Theorem 6 of [43]) showed that $\mathcal{SL} \subseteq \mathcal{S}_3$.

The following is equivalent to a result of Tutte [138], it does not assume the Four Color Theorem [3], and it is analogous to Theorem 2.4:

Theorem 7.1. For any planar graph G with planar dual G^* , $G \in \mathcal{S}_3$ if and only if $\chi(G^*) \leq 4$. ■

There is a reduction method for determining membership in \mathcal{S}_3 similar to the reduction method for \mathcal{SL} . Here is the analogue of Theorem 3.1:

Theorem 7.2 [40]. Let G be a graph and let H be a subgraph of G . If $H \in \mathcal{CL}$ or if H is a 4-cycle, then

$$G \in \mathcal{S}_3 \iff G/H \in \mathcal{S}_3. \quad \blacksquare$$

If G is planar, then contraction of a face in G is equivalent to deletion of a vertex in the planar dual G^* . This observation and Theorems 7.1 and 7.2 (with $H = C_4$) imply that a smallest counterexample G^* to the Four Color Conjecture would have $\delta(G^*) \geq 5$. This result is Theorem 6.4.4 in [116], and it also follows from the more general Theorem 11.2.6 of Dirac in [116].

Another reduction method for determining membership in \mathcal{S}_3 involves the lifting of edges. Let G be a graph, let $v \in V(G)$, and let uv and vw be distinct edges incident with v . Define $G(uv, vw)$ to be the graph obtained from $G - \{uv, vw\}$ by adding the new edge uw . The first part of the following result follows from a result of Fleischner [66] (and it follows from Mader's Lifting Theorem [107]):

Theorem 7.3. For any nontrivial graph G with no cut edge, and for any vertex $v \in V(G)$ with $d(v) \neq 3$, there are incident edges uv and vw in $E(G)$ such that $G(uv, vw)$ has no cut edge. Furthermore, $G \in \mathcal{S}_3$ if $G(uv, vw) \in \mathcal{S}_3$.

Suppose G is a graph that is not an even graph and that G has no cut edge. Repeated applications of Theorem 7.3 will dissolve all vertices of even degree and convert G into a 3-regular graph of order $O(G)$ that is in \mathcal{S}_3 only if $G \in \mathcal{S}_3$. The latter part of Theorem 7.3 holds for various other families of graphs defined in terms of cycle double covers (see [87]), but not for \mathcal{SL} .

Tutte [140] (also [141]) and Matthews [109] conjectured that if $G \notin \mathcal{S}_3$ and if G has no cut edge, then some subgraph of G is contractible to the Petersen graph. Alspach and Zhang [2] showed that a 3-regular graph has a cycle double cover if it has no subgraph contractible to the Petersen graph.

Let G be a graph, let $k \geq 2$, and assign a direction to each edge of G . A *nowhere zero k-flow* is an assignment of nonzero members of \mathbf{Z}_k to $E(G)$ such that at each vertex, the sum of the weights on the incoming edges equals the sum of the weights on the outgoing edges. Clearly, G has a nowhere zero 2-flow if and only if G is even. It follows from the equivalence (i) \iff (ii) of Theorem 5.5 of [88] that $G \in \mathcal{S}_3$ if and only if G has a nowhere zero 4-flow. If G has a nowhere zero k -flow, for some k , then G has a nowhere zero $(k + 1)$ -flow. The Petersen graph has a nowhere zero 5-flow, but no nowhere zero 4-flow. Seymour [127] showed that any graph with no cut edge has a nowhere zero 6-flow, but it is not known if all such graphs have a nowhere zero 5-flow.

Jaeger [86] proved that any graph with no cut edge and no 3-cut has a nowhere zero 4-flow (i.e., it lies in \mathcal{S}_3), and Tutte ([22], unsolved problem 48) conjectured that such a graph has a nowhere zero 3-flow. (A 3-cut is an edge cut of three edges.) Using Theorem 7.2, Catlin generalized Jaeger's result in [40] and it can be restated as follows:

Theorem 7.4. If a graph G with no cut edge has at most ten 3-cuts, then exactly one of these holds:

- (a) $G \in \mathcal{S}_3$;
- (b) G is contractible to the Petersen graph. ■

The hypothesis "at most ten" (in Theorem 7.4) can be improved, but a result of this sort in [43] has a gap in its proof.

Consider a graph G embedded on a surface. A proper coloring of the faces is called *packed* if at every vertex of degree at least 3, the incident faces have been assigned at least three different colors, and if each edge is incident with faces of two different colors. If G is a 3-regular graph embedded on some surface, then any proper coloring must be packed.

Let G be a graph. A subgraph H of G *evenly spans* G if

- (i) each vertex of G is of even degree in H ;
- (ii) each vertex of degree at least 3 has nonzero degree in H ; and
- (iii) each component of H contains evenly many members of $O(G)$.

Call a graph G *evenly spanned* if some subgraph H exists that evenly spans G . Let \mathcal{ES} denote the family of graphs that are evenly spanned. Of course, the Petersen graph is not evenly spanned. Archdeacon [4] proved $\mathcal{SL} \subset \mathcal{ES}$ for graphs with minimum degree at least 3.

Let G be a graph. A *3-splitting* of G is any 3-regular graph H that can be converted to G by a sequence of edge contractions and subdivisions.

Theorem 7.5 ([4], [16], [42], [87]). Let G be a graph. These are equivalent:

- (a) $G \in \mathcal{ES}$;
- (b) G has a 3-splitting H with $\chi'(H) = 3$;
- (c) G embeds on some orientable surface with a packed 4-coloring of the faces;
- (d) G embeds on some surface with a packed 4-coloring of the faces;
- (e) G embeds on some surface with a packed 3-coloring of the faces;
- (f) $G \in \mathcal{S}_3$;
- (g) G has a double cover by at most 4 even graphs. ■

A proof of the equivalence of conditions (a)–(e) of Theorem 7.5 was given by Archdeacon [4] for any graph G with $\delta(G) \geq 3$, and he noted that Tutte [137] had obtained that result for 3-regular graphs. The hypothesis $\delta(G) \geq 3$ was removed in [42], where (f) was added to the list. The equiva-

lence of (f) and (g) was proved in [16] (Prop. 4) and also in Prop. 3 of [87]. Other conditions equivalent to those of Theorem 7.5 can be found in [87] and [88].

8. OTHER REMARKS

We have not reviewed related literature regarding directed graphs, matroids, the Chinese postman's problem, eulerian graphs, the interval number, cycle decompositions, minimum covers, flows, and most of the literature on cycle double covers not concerning \mathcal{S}_3 . Some related literature involves embeddings of graphs on surfaces. Jaeger [87] reviewed various methods to attack the cycle double cover conjecture, and he reviewed flow problems in [88]. Fleischner [68] reviewed work on cycle decompositions, and he has discussed problems involving decompositions of and transitions in eulerian graphs [69]. For more references concerning hamiltonian 3-regular graphs, see [84]. Research on snarks (graphs not in \mathcal{S}_3 that are minimal in a certain sense) is discussed in [60] and [71]. The reduction method for \mathcal{SL} and \mathcal{S}_3 has been generalized to other families of graphs (see [39], and for a partial summary, see [38] or [46]).

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